A High-Level Analysis of SpaceX's Upcoming Starship and Super Heavy Vehicles

William "Cody" Strong[1]

The University of Alabama, Tuscaloosa, Alabama, 35487, United States

SpaceX is currently developing their latest launch vehicle system, Starship, designed to open up the access to space by employing technologies that allow low cost full and rapid reuse of the launch vehicles. Launch, orbit refueling, and landing are three key aspects to missions involving this future reusable launch vehicle, and in this report I analyze the potential performance capabilities of Starship at a high level in these three major situations. For the launch of Starship and Super Heavy, I found that the vehicle would not reach the advertised 500 km orbit with a full 100 Mg of payload mass, nor could it reach geosynchronous transfer orbit using my model. However, this discrepancy is possibly a result my of overly conservative estimates when sourcing data from unofficial sources and compounding over successive calculations. I found for orbital refueling that Starship could indeed make a maneuver to Mars with just three to five orbital refueling missions. Lastly, my landing analysis concurs with the, admittedly limited, real world data on descent time, altitude at engine ignition, and thrust output.

I. Nomenclature

A. Acronyms and Terminology

CEA	=	Chemical Equilibrium with Applications; NASA software program for engine analysis
CH_4	=	chemical formula for methane
CRS	=	Commercial Resupply Services
delta-V	=	change in velocity; a measure of how much energy a rocket has available for maneuvers
FCC	=	Federal Communications Commission
GEO	=	geosynchronous equatorial orbit
GTO	=	geosynchronous transfer orbit
FFSC	=	full-flow stage combustion
KSC	=	Kennedy Space Center
LEO	=	low Earth orbit
LOX	=	liquid oxygen
MECO	=	main engine cutoff
methalox	=	rocket bipropellant composing of liquid methane as a fuel and liquid oxygen as an oxidizer
RTLS	=	return to launch site
SH	=	Super Heavy
SN	=	serial number (e.g., Starship SN 15)
SS	=	Starship (the vehicle)
SSO	=	sun synchronous orbit
TVC	=	thrust vector control
TPS	=	thermal protection system
TWR	=	thrust-to-weight ratio

[1] Graduate student, Department of Aerospace Engineering and Mechanics.

B. Equation Variables

A	=	area (m^2)
a	=	semi-major axis (km)
C_3	=	trajectory characteristic energy
C_D	=	coefficient of drag
c	=	effective exhaust velocity (m/s)
$c^\star$	=	characteristic exhaust velocity (m/s)
D	=	diameter (m)
F	=	force (N) or (kN)
F_{inert}	=	inert mass fraction
$\hbar$	=	height (m)
h	=	altitude (km) or (m)
I_{sp}	=	specific impulse (s)
L	=	characteristic length (used with Re)
$\mathcal{L}$	=	landing propellant reserve percentage
m	=	mass (kg) or (Mg)
$\dot{m}$	=	mass flow rate (kg/s) or (Mg/s)
n	=	number
O/F	=	oxidizer–fuel ratio
P	=	pressure (Pa)
r	=	current orbit radius (constant for circular orbits) (km)
Re	=	Reynolds number
t	=	time (s)
$\mathbb{t}$	=	engine throttle position (40% to 100%)
v	=	velocity (m/s)
$\mathbb{V}$	=	volume (m^3)
α	=	azimuth angle
Δv	=	delta-V; change in velocity (m/s)
ρ	=	density (kg/m^3)
λ	=	nozzle efficiency
$\imath$	=	inclination (deg.)
ϕ	=	latitude of the launch site (deg.)

C. Constants

g_0	=	standard gravity acceleration (9.80665 m/s^2) [1]
$\mathcal{M}$	=	dynamic viscosity of the air at sea level (1.789×10^{-5} kg/m/s) [1]
R_{Earth}	=	equatorial radius of Earth (6.378×10^6 m) [2]
μ	=	gravitational parameter of Earth (3.986×10^{14} $m^3 s^{-2}$) [2]

II. Introduction

SPACEX's upcoming Starship System is poised to take the rocket launch market by storm. The system promises frequent, reliable, and low-cost access to space. Although the Starship System will end up being the largest and most powerful launch vehicle in the world, because it is designed for full and rapid reusability, Starship is intended to also become the lowest priced access to space [3].

"Starship" is the name given to both SpaceX's next generation launch system as a whole and the second stage vehicle as well. For the purposes of this report, "Starship" when used alone will refer to the singular vehicle. However, to avoid confusion, I will typically clarify my meaning such as "Starship System" or "Starship vehicle" or I will avoid the problem altogether opting for "vehicle," "spacecraft," "second stage," or something of that sort.

The Super Heavy booster, when it becomes operational, will hold the record for the largest number of simultaneously running engines on a launch vehicle. Currently, the reigning champion for this feat is SpaceX's own heavy-lift rocket, the aptly named Falcon Heavy, with 27 simultaneous engines [4]. Anywhere from 28 to 31 full-flow staged combustion (FFSC) Raptor engines will give this nine-meter diameter stainless-steel behemoth its approximately 72 meganewtons of thrust at liftoff [3, 5, 6]. Together with the Starship second stage, up to 100 Mg can allegedly be put into orbit [7].

Whereas Super Heavy is designed only to be used on Earth, the Starship vehicle is designed to operate under a variety of conditions and locations across the solar system. For atmospheric entry, Starship will position itself nearly perpendicular to the direction of airflow, using a massive heat shield on the large ventral cross-section of the vehicle. This operation differs substantially from the atmospheric entry of a traditional lifting body where the spacecraft is oriented with a much shallower angle of attack. A total of four actuating flaps will provide control authority to the spacecraft using differential drag as it enters the atmosphere. Once the spacecraft is subsonic and has removed nearly all horizontal velocity, Starship will orient itself horizontal with respect to the ground to begin falling like a skydiver (Figure 1). The flaps continue to provide control authority using differential drag. Once Starship has neared the landing pad, it will relight its Raptor engines to reorient itself vertically and perform a propulsive landing on the landing pad [3, 8].

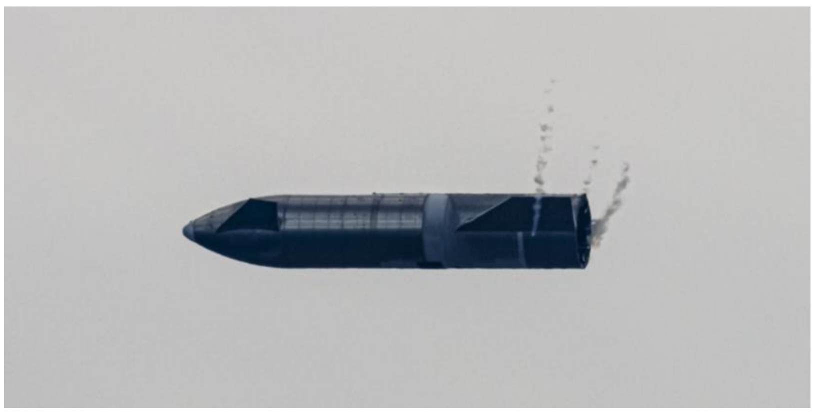

Figure 1. Starship horizontal descent

Starship being designed for reusability is constructed with components that negatively impact performance from a mass standpoint, such as landing legs, a thermal protection system (TPS), non-jettisonable fairing, aerodynamic control surfaces, and extra engines for operating at different atmospheric pressures just to name a few [3, 8]. Mass dedicated to recovery hardware can become a problem because, due to the physics of trying to reach orbit, every unit of mass on your final rocket stage that does not contribute to your delta-V (structural mass, avionics, anything that is not propellant, etc.) is a unit of mass of payload that you are not able to place into orbit. This problem became a clear problem for the Space Shuttle and the Orbiter never traveled beyond low Earth orbit (LEO) [9]. SpaceX intends to remedy this problem by refueling the Starship second stage in orbit effectively “resetting” the rocket equation and allowing the capability for Starship to perform missions that require a high amount of delta-V [3, 8].

III. Analysis

A. Raptor Engine

1. Background

At the heart of Starship lies the Raptor engine. When designing a launch vehicle, the design requirements of the engine will typically dictate much of the design of the rest of the launch vehicle, usually in the way of propellant tank dimensions and the structure bearing the primary thrust loads [10]. SpaceX began development of the Raptor engine in 2012, nearly a decade ago [11]. Originally designed to burn liquid hydrogen and liquid oxygen, the fuel was quickly swapped for liquid methane. Liquid methane is more dense than liquid hydrogen allowing the structural mass of the rocket to be much lower. Like hydrogen, methane can be produced on the surface of Mars—an important factor that helps SpaceX meet its goals of making routine trips to the Red Planet [12].

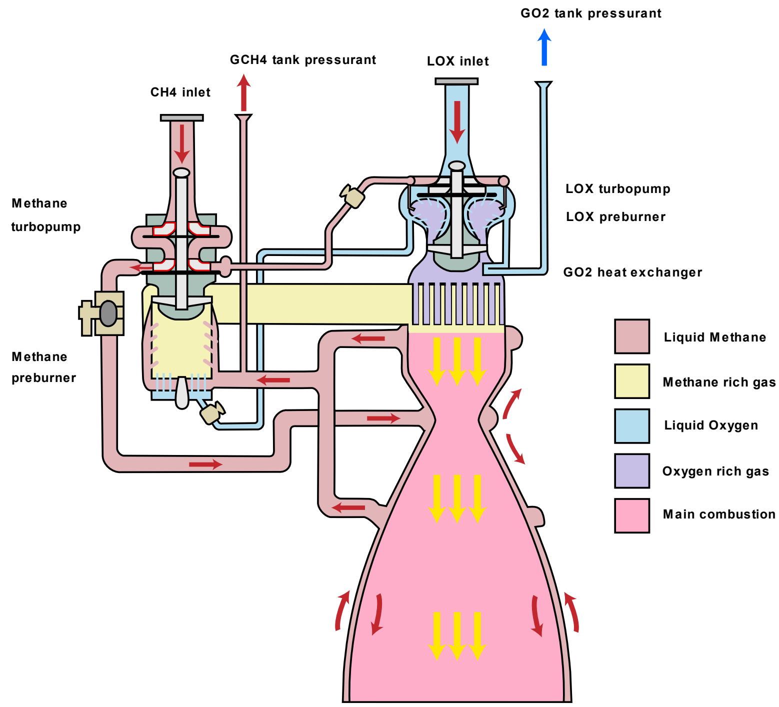

Figure 2. Unofficial Raptor engine schematic to illustrate the full flow staged combustion cycle. Courtesy of NASASpaceFlight.com forum user HVM. [13]

Raptor is a full-flow stage combustion (FFSC) methalox engine. "Staged" refers to the propellant being partially combusted in a pre-burner to drive the turbopumps before the propellant enters the main combustion chamber. "Full-flow" staged combustion engines are a particular type of staged combustion engine that have two separate pre-burners—one oxygen rich and one fuel rich—each being used to drive separate turbines, shafts, and turbopumps. Figure 2 helps to illustrate these concepts. This type of engine has performance advantages over standard staged combustion engines, primarily because the partially combusted exhaust products of the oxygen and fuel rich pre-burners can be combined and combusted together in the main combustion chamber slightly adding to the performance. However, designing a FFSC engine can be challenging as the hot oxygen rich gas from the oxygen rich pre-burner will readily break down most metals, which could be a great concern considering SpaceX is aiming to have the Raptor engines reused over 1000 times [14]. Additionally, since FFSC engines have two separate turbopumps, keeping the oxidizer/fuel (O/F) ratio constant and preventing combustion instabilities can be difficult when significantly throttling the engine, a vital part of performing a gentle landing. After many years of subscale static testing, on 25 July 2019 the Raptor engine became the first full-flow staged combustion engine to take flight [15]. The design has come quite close to that of the full production engine, and as of the end of April 2021, at least 66 flight-ready Raptor engines had been built, individually tested, and later mounted to Starships and tested again [16]. Many of these engines were able to perform high altitude flight tests with Starship prototypes SN8 through SN11 and SN15, but so far, the three Raptor engines mounted on SN15 have been the only ones to return to Earth safely [16].

2. CEA Analysis

Before being able to analyze the performance of the Super Heavy and Starship launch vehicles, I must be able to model the engine mathematically, allowing me to consider how the engine performance changes with altitude. Finding accurate and complete specifications for the raptor engine was a bit difficult, likely due to it still being in development. One surprising place to find data is Elon Musk's Twitter account as the SpaceX CEO is typically forthcoming with information about Starship and Raptor development. However, information can change significantly from week to week, and not all information about Raptor from this source is the same age, so it is difficult to build a complete picture. I was eventually able to find a mostly complete set of specifications for Raptor was compiled in an environmental assessment for Starship at Kennedy Space Center [17]. Though the information is slightly outdated, it is surprisingly complete, and I will use the data for the analysis of the engines and subsequent performance calculations for the launch vehicles. Raptor specifications from the environmental assessment and other sources are compiled in

Table 1. Equation 1 was used to fill in any remaining data, where D is the diameter of the circular cross section and A is the area.

Table 1. Raptor engine specifications

Common	Throat diameter	D_t	0.222	m	[17]
	Throat area	A_t	0.03857	m^2	Equation 1
	Combustion chamber pressure	P_c	30	MPa	[18]
	Oxidizer–fuel ratio	O/F	3.55		[17]
Sea Level Optimized	Nozzle exit diameter	$D_{e_{SL}}$	1.3	m	[17]
	Nozzle exit area	$A_{e_{SL}}$	1.327	m^2	Equation 1
	Exit to throat area ratio	$A_{e_{SL}}/A_t$	34.34		[17]
Vacuum Optimized	Nozzle exit diameter	$D_{e_{Vac}}$	2.8	m	[19]
	Nozzle exit area	$A_{e_{Vac}}$	6.158	m^2	Equation 1
	Exit to throat area ratio	$A_{e_{Vac}}/A_t$	159.7		

$$A = \frac{\pi D^2}{4} \qquad \text{Equation 1}$$

I used NASA's Chemical Equilibrium with Applications (CEA) program to model the combustion effects of the engine inputting the Raptor specifications as necessary [20]. For the propellant, I used methane and oxygen estimating the temperatures to be 200 K since they enter the combustion chamber in the gas phase [21]. Frozen composition was assumed in the calculations since the exhaust quickly cools as it expands in the nozzle. Compared with equilibrium flow which overestimates performance, an analysis performed with frozen flow, such as what I have done here, will slightly underestimate engine performance [22].

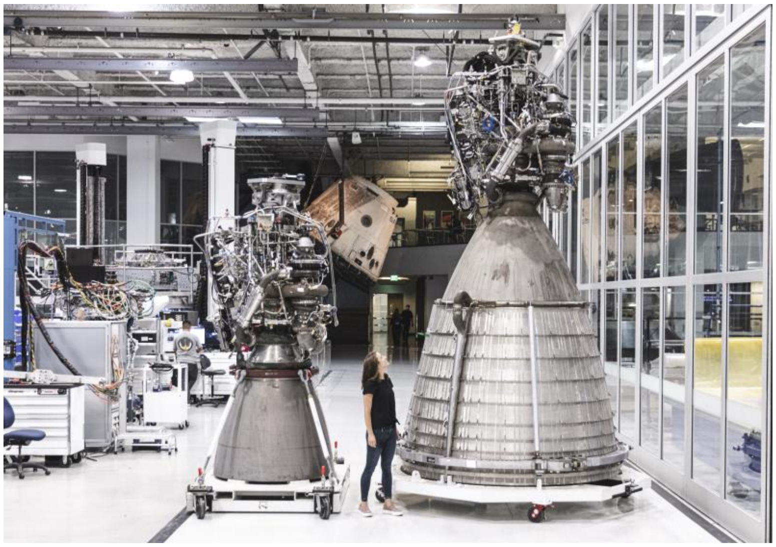

Figure 3. Raptor engine optimized for sea level operation (left) and vacuum operation (right) have the same throat diameter but vastly different nozzle diameters. Courtesy of SpaceX [23]

Table 2. Raptor performance results from CEA [20]

Sea Level Optimized	Exhaust to combustion chamber pressure ratio	$P_{e_{SL}}/P_c$	$1/421.32$	
	Exhaust gas density	$\rho_{e_{SL}}$	0.1434	kg/m^3
	Exhaust velocity	v_e	3292.1	m/s
	Characteristic exhaust velocity	c⋆	1846.7	m/s
Vacuum Optimized	Exhaust to combustion chamber pressure ratio	$P_{e_{Vac}}/P_c$	$1/3184.0$	
	Exhaust gas density	$\rho_{e_{Vac}}$	0.028797	kg/m^3
	Exhaust velocity	v_e	3533.6	m/s
	Characteristic exhaust velocity	c⋆	1846.7	m/s

Mass flow rate of propellant through the engine ($\dot{m}$) is calculated to be $\dot{m} = 626.5\ kg/s$ (Equation 2). This value is the same for both the sea level and vacuum engines since the combustion chamber is the same for both engines. Then, Equation 3 is used to determine the effective exhaust velocity (c) as a function of atmospheric pressure. Having the effective exhaust velocity available as a function of atmospheric pressure is of particular importance for the Super Heavy vehicle because the constantly changing atmospheric conditions as the booster ascends have a tangible effect on the performance of the engines. The effective exhaust velocity function is then used in Equation 4—where $g_0 = 9.8066\, m/s^2$ is the standard gravity—to find specific impulse (I_{sp}) as a function of atmospheric pressure, which is used to find the thrust force as a function of atmospheric pressure (Equation 5). The nozzle efficiency (λ) in Equation 4 is estimated to be 98% efficient [10].

$$c^\star = \frac{A_t P_c}{\dot{m}} \qquad \text{Equation 2}$$

$$c(P_{atm}) = v_e + \frac{A_e\left(P_c\, {P_e}/{P_c} - P_{atm}\right)}{\dot{m}} \qquad \text{Equation 3}$$

$$I_{sp}(P_{atm}) = \lambda\, \frac{c(P_{atm})}{g_0} \qquad \text{Equation 4}$$

$$F_T(P_{atm}) = I_{sp}(P_{atm})\, g_0\, \dot{m} \qquad \text{Equation 5}$$

Table 3 compares the specific impulse (Equation 4) and thrust force (Equation 5) for the sea level Raptor engine at sea level atmospheric pressure (101.3 kPa) and in vacuum and the vacuum optimized engine in a vacuum. Going a step further, I plotted the performance of the sea level engine at various altitudes from 0 km to 75 km (Figure 4). The pressure data at the range of altitudes was sourced from the “atmoscoesa” function in MATLAB which uses the 1976 U.S. Standard Atmosphere model [1]. The specific impulse and thrust force curves in Figure 4 are proportional and thus have the same shape. The main takeaway from Table 3 and Figure 4 is that the specific impulse for the vacuum optimized engine operating in a vacuum is larger than the specific impulse of the sea level engine operating in a vacuum which is in turn greater than the sea level Raptor operating at sea level atmospheric pressures. The same is true for the thrust force. Starship will have three vacuum Raptor engines to use in most maneuvers, but when landing back on Earth, Starship will use its three sea level Raptors instead. Even on Mars and other planetary bodies, Starship will use the sea level Raptors for landing, not because of efficiency, but because these three engines have thrust vector control (TVC) system for steering, but the vacuum engines do not have this feature and the nozzles will be fixed to the airframe [24].

Table 3. Calculated specific impulse and thrust force for sea level and vacuum optimized Raptor engines

	P_{atm} [kPa]	$I_{sp}(P_{atm})$ [s]	$F_T(P_{atm})$ [kN]
Sea Level Optimized	101.3	329	2030
	0	351	2160
Vacuum Optimized	0	370	2270

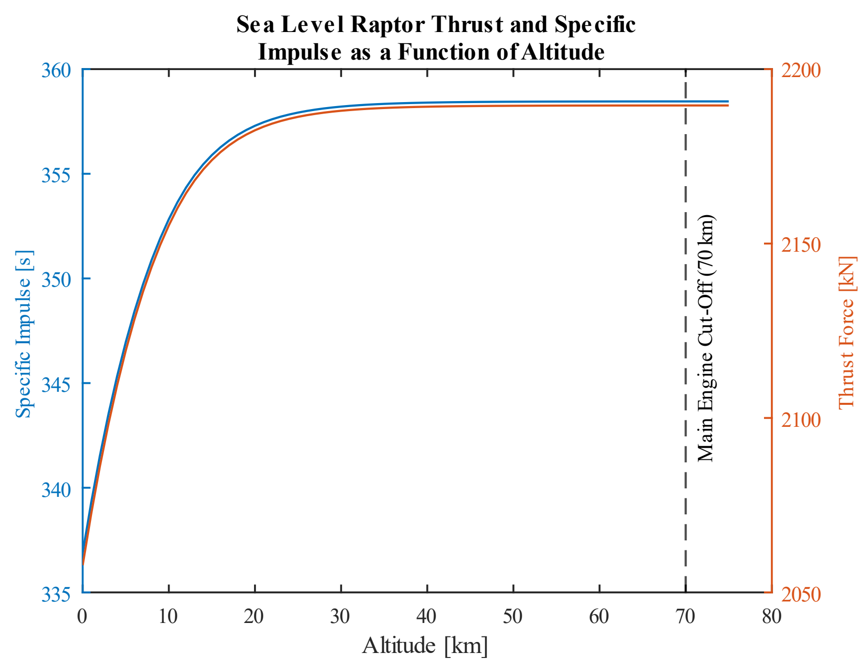

Figure 4. Sea level Raptor thrust and specific impulse as a function of altitude

B. Vehicle Specifications

The specifications of the Starship and Super Heavy vehicles must be known, or realistically approximated, to construct models of the flight profiles. Furthermore, the vehicle data must be somewhat accurate for the results of the models to be useful. This aspect proved to be somewhat challenging as the Starship development program has been in constant flux since its beginning and designs change frequently.

Both SpaceX as a company and its CEO, Elon Musk, have been quite transparent regarding details on the Starship program. All Starship test launches are broadcast live by SpaceX, and Musk will typically give a presentation on the progress of Starship's development about once a year. More frequently Musk will often provide updates through his Twitter. Building and testing Starship prototype vehicles out in an open field—effectively the polar opposite of the large cleanrooms of typical aerospace companies—SpaceX has garnered attention from "fans" of space and the general public alike. Some fans go as far as setting up cameras to livestream development progress over the internet twenty-four hours per day. With all this development happening quite literally out in the open, it is no surprise that dozens of websites, video livestreams, and Internet forums have sprung up allowing people to post progress updates of Starship development almost constantly each day. While fan photos and Musk's Tweets may be unconventional sources of information for a typical research report, they have been invaluable in understanding and analyzing the Starship program.

Almost all Starship and Super Heavy vehicle specifications required for the calculations in following sections are included in Table 4 and Table 5, respectively. Many parameters were sourced directly, either Tweets from Musk or the 2019 Starship Update presentation, but there were some that had no primary source and were calculated from other parameters. Much more information is known about the Starship vehicle than Super Heavy, which needed some assumptions to be made before all the blanks could be filled in. First, the dry mass of the vehicle is still not officially known. To calculate an approximate value for Super Heavy's dry mass, I assumed that the inert mass fraction was equal to the inert mass fraction of Starship (Equation 7). This value is likely to be somewhat conservative since the additional hardware on Starship to enable reusability—flaps, heat shield, permanent fairing, etc.—increases its inert mass fraction. But according to Ref. [10], even an inert mass fraction of 0.091 is on the low end of launch vehicles with typical values ranging from 0.08 to 0.7. The dry mass is finally calculated using Equation 8.

Table 4. Starship Specifications

Parameter	Symbol	Value	Units	Source
Diameter	D^{SS}	9	m	[3]
Height	h^{SS}	50	m	[3]
Dry mass	$\mathrm{m}_{\mathrm{dry}}^{SS}$	120	Mg	[3]
Total propellant mass	$\mathrm{m}_{\mathrm{prop}}^{SS}$	1200	Mg	[3]
Header propellant mass	$m_{headerProp}$	30	Mg	[25]
Main propellant mass	$\mathrm{m}_{\mathrm{mainProp}}^{SS}$	1170	Mg	Equation 6
Inert mass fraction	$\mathcal{F}_{inert}^{SS}$	0.091	-	Equation 7
Number Raptor sea level engines	n_{eng}^{SL}	3	-	[3]
Number Raptor vacuum engines	n_{eng}^{vac}	3	-	[3]

$$\mathrm{m}_{\mathrm{mainProp}}^{\mathrm{SS}} = \mathrm{m}_{\mathrm{prop}}^{\mathrm{SS}} - \mathrm{m}_{\mathrm{headerProp}} \quad \text{Equation 6}$$

$$\mathcal{F}_{\mathrm{inert}}^{\mathrm{SS}} = \frac{\mathrm{m}_{\mathrm{dry}}}{\mathrm{m}_{\mathrm{prop}} + \mathrm{m}_{\mathrm{dry}}}$$

Assume:

$$\mathcal{F}_{\mathrm{inert}}^{\mathrm{SH}} = \mathcal{F}_{\mathrm{inert}}^{\mathrm{SS}} \quad \text{Equation 7}$$

Second, the amount of propellant Super Heavy required in reserve for a landing was also not known. Super Heavy will always perform a return to launch site (RTLS) landing instead of sometimes landing downrange on a drone ship like the first stage of Falcon 9 [7]. The RTLS maneuver requires an extra startup of the engines, thus requiring more fuel, than a drone ship landing. However, since the stainless-steel construction of Super Heavy can withstand higher temperatures, SpaceX does not intend the booster to fire its engines as it re-enters the thickest part of the atmosphere like Falcon 9 [3]. Ultimately, I assumed that the percentage of total propellant required to be in reserve for landing Super Heavy was equal to the percentage similarly required by the Falcon 9 first stage booster. Ref. [26] estimates this margin to be five percent of the total propellant mass. Using this value for Super Heavy, like the value for inert mass fraction previously, is potentially a conservative estimate; since the inert mass increases with the surface area of the vehicle (length squared), but propellant mass increases with the volume of the vehicle (length cubed), the larger vehicle should have a lower inert mass ratio, and therefore higher delta-V, than the smaller vehicle for the same percentage of total propellant.

Table 5. Super Heavy Specifications

Parameter	Symbol	Value	Units	Source
Diameter	D^{SH}	9	m	[3]
Height	$\hbar^{SH}$	70	m	[6]
Dry mass	$\mathrm{m}_{\mathrm{dry}}^{SH}$	340	Mg	Equation 8
Max propellant mass	$\mathrm{m}_{\mathrm{prop}}^{SH}$	3400	Mg	[6]
Landing propellant reserve percent	$\mathcal{L}$	5%	-	[26]
Mass reserve propellant	$\mathrm{m}_{\mathrm{reserveProp}}$	170	Mg	Equation 9
Main propellant mass	$\mathrm{m}_{\mathrm{mainProp}}^{SH}$	3230	Mg	Equation 10
Inert mass fraction	$\mathcal{F}_{inert}^{SH}$	0.091	-	Equation 7
Number Raptor sea level engines	n_{eng}^{SL}	31	-	[17]
Number Raptor vacuum engines	n_{eng}^{vac}	0	-	[17]

$$\mathrm{m}_{\mathrm{dry}} = \frac{\mathcal{F}_{\mathrm{inert}}}{1 - \mathcal{F}_{\mathrm{inert}}} \mathrm{m}_{\mathrm{prop}} \qquad \text{Equation 8}$$

$$\mathrm{m}_{\mathrm{reserveProp}} = \mathcal{L}\, \mathrm{m}_{\mathrm{prop}} \qquad \text{Equation 9}$$

$$\mathrm{m}_{\mathrm{mainProp}}^{\mathrm{SH}} = \mathrm{m}_{\mathrm{prop}}^{\mathrm{SH}} - \mathrm{m}_{\mathrm{reserveProp}} \qquad \text{Equation 10}$$

C. Launch

1. Background

Starship and Super Heavy are currently intended to be launched from two locations: Kennedy Space Center (KSC) in Florida and Boca Chica, Texas, situated next to the Rio Grande and the Gulf of Mexico. Construction progress is halted currently on the launch pad at Launch Complex 39A in KSC where SpaceX Falcon 9 and Falcon Heavy rockets currently lift off. But at Boca Chica—where Starship and Super Heavy prototypes are being built and tested—construction on the pad for orbital launches is moving along full steam ahead. Presently, crews are working to assemble the hexagonal "launch table," dubbed such because once assembled, the structure will be transported down the road to the launch site where it will be hoisted atop six concrete and steel columns jutting out of the ground and slightly canted inwards [27]. Once in place, it will like somewhat like a (very large) table, though I prefer to think of it more as a milk stool. Meanwhile, adjacent to the launch pad the launch tower is being built. This steel trussed structure will feature a large crane to hoist the Super Heavy booster into place on top of the launch pad as well as Starship on top of Super Heavy. The tower will also serve as crew access to the Starship when the time comes for crewed launches [27].

SpaceX is ambitiously targeting an orbital launch of a Starship prototype as soon as July [28, 27] , but until the launch occurs and real-world data can be gathered, can the performance of such a launch be estimated? How much payload can it put into which kinds of orbits?

2. Low Earth Orbit (LEO)

Starship will be quite active in LEO deploying vast mega-constellations like SpaceX's own Starlink communications network, launching large space telescopes, and resupplying the ISS and future commercial space stations. SpaceX currently advertises an incredible "100+" Mg of payload to LEO in a single launch. This service is available for circular orbits up to 500 km in altitude with an inclination up to 98.9 degrees [7]. I will use this reference orbit to compare the advertised payload mass to orbit with the values produced by my model.

The amount of delta-V Starship can provide is a relatively straightforward calculation. Since the vehicle only operates in a vacuum for launches, a constant value for the effective exhaust velocity (c) may be used. First, the mass of Starship fully loaded is calculated (m_0) in Equation 11. Then, in Equation 12 the mass after the maneuver is calculated. With both the before and after masses calculated, the total delta-V available to Starship (Δv_{SS}) can be calculated using Equation 13.

$$m_i = m_{mainProp} + m_{headerProp} + m_{dry} + m_{payload} \quad \text{Equation 11}$$

$$m_f = m_{headerProp} + m_{dry} + m_{payload} = m_i - m_{mainProp} \quad \text{Equation 12}$$

$$\Delta v_{SS} = -c \ln\left(\frac{m_f}{m_i}\right) \quad \text{Equation 13}$$

where $m_{payload}$ = payload mass in Starship (could be zero)
Δv_{SS} = total delta-V available from Starship
c = exhaust velocity (found in III.A.2)

Now, the delta-V imparted by Super Heavy must be determined. The tricky part about Super Heavy is that it operates in the thick, sea level part of the atmosphere as well as the rarified parts of the upper atmosphere. This range of operating altitudes means that the exhaust velocity of the engines will not be constant throughout the duration of the flight. The simplest approximation would be to simply take the average of the exhaust velocities at sea level and in a vacuum. However, this method does not account for changes to the ascent profile; a shallower ascent will spend more time in the thicker part of the atmosphere, reducing engine performance, but a steeper ascent has its own trade-offs. Trajectory design is not itself a part of this investigation, so I will instead apply the launch trajectory of the Falcon 9 during the Commercial Resupply Services 18 (CRS-18) mission [29]. This trajectory provides an altitude as a function of time. The altitude as a function of altitude is numerically integrated (Equation 15) from liftoff until t_f (Equation 14) when main engine cutoff (MECO) occurs, obtaining a mean altitude ($\bar{h}$) at which to calculate the atmosphere dependent parameters of the Raptor engines (Equation 16), where $P_{atm}(\bar{h})$ is the atmospheric pressure as a function of altitude. Initial and final masses of Super Heavy, m_0 (Equation 17) and m_1 (Equation 18), respectively, are calculated similarly to the respective values for Starship. Finally, the delta-V available from Super Heavy (Δv_{SH}) is calculated (Equation 19).

$$t_f = \frac{m_{mainProp}}{n_{eng}\, \dot{m}} \quad \text{Equation 14}$$

$$\bar{h} = \frac{1}{t_f}\int_0^{t_f} h\, dt \quad \text{Equation 15}$$

$$\bar{c} = c\left(P_{atm}(\bar{h})\right) \quad \text{Equation 16}$$

$$m_i = m_{mainProp} + m_{reserveProp} + m_{dry} + m_{SS} \quad \text{Equation 17}$$

$$m_f = m_{reserveProp} + m_{dry} + m_{SS} = m_i - m_{mainProp} \quad \text{Equation 18}$$

$$\Delta v_{SH} = -\bar{c} \ln\left(\frac{m_f}{m_i}\right) \quad \text{Equation 19}$$

where t_f = the moment when Super Heavy has run out of its main propellant
n_{eng} = number of sea level Raptor engines on the Super Heavy vehicle
h = current altitude of Super Heavy within the atmosphere (not to be confused with h_{orb})
$P_{atm}(h)$ = atmospheric pressure at altitude h
$c(P_{atm})$ = effective exhaust velocity at pressure P_{atm}
$\bar{c}$ = effective exhaust velocity at the rocket's mean altitude
m_{SS} = total mass of Starship calculated in Equation 11
Δv_{SH} = total delta-V available from Super Heavy

Calculation results of mass independent parameters for this flight profile are included in Table 6 and are likely similar to other flight profiles. In particular, the MECO time was calculated to be 166 seconds. A recent Federal Communications Commission (FCC) filing for SpaceX's upcoming first orbital test flight of Starship and Super Heavy indicates MECO occurring at 169 seconds [30]. I believe my calculation is close enough to verify its legitimacy, and the discrepancy is likely due in part to Super Heavy throttling down its engines during the period of maximum aerodynamic pressure which I did not consider in my calculations.

Table 6. Super Heavy performance

Name	Symbol	Value	Unit	Source
MECO	t_f	166	s	Equation 14
Mean booster altitude	$\bar{h}$	26.8	km	Equation 15
Atmospheric pressure (mean altitude)	P_{atm}	1893	Pa	[1]
Effective exhaust velocity (mean altitude)	$\bar{c}$	3439	m/s	Equation 16

The total amount of delta-V available to the Starship Launch System (ΔV_{avail}) from Equation 20 dictates which orbits are accessible in a single launch. Defined in Equation 25, the total amount of delta-V required to reach a particular orbit (ΔV_{req}) is the sum of the target orbital velocity (V_{orbit}), the delta-V corresponding to the change in gravitational potential energy (ΔV_{poten}), the delt-V gained by or required to overcome the rotation of the Earth (ΔV_{rot}), and lastly the delta-V attributed to aerodynamic drag, steering, and gravity losses (ΔV_{losses}). For this analysis, all launches will enter a circular orbit as reflected by Equation 21. The delta-V due to the change in potential energy (Equation 22) can be imagined as a transfer orbit between the target orbit and a reference orbit defined at the surface of the Earth all minus the velocity of the target orbit. The delta-V from Earth's rotation is a function of launch site latitude (ϕ) and launch azimuth (α), which is in-turn a function of launch site latitude and orbit inclination (i). Typically, launches take advantage of the rotation of the Earth, so this fact is reflected in Equation 24 with a negative sign. Lastly, using data from historical rockets, the delta-V from losses can be estimated as about $1700\ {}^{m}/_{s}$ [10]. Setting (ΔV_{avail}) equal to (ΔV_{req}), substituting in the appropriate equations and values into Equation 25, and rearranging for orbit altitude (h_{orb}) yields (Equation 26), the value of orbit altitude as a function of available delta-V. This value of orbit altitude is the maximum orbit altitude achievable (for a circular orbit) for a given delta-V input.

$$\Delta v_{avail} = \Delta v_{SH} + \Delta v_{SS} \quad \text{Equation 20}$$

$$v_{orbit} = \sqrt{\frac{\mu}{R_{Earth} + h_{orb}}} \quad \text{Equation 21}$$

$$\Delta v_{poten} = \sqrt{\frac{2\mu}{R_{Earth}} - \frac{\mu}{R_{Earth} + h_{orb}}} - \sqrt{\frac{\mu}{R_{Earth} + h_{orb}}} \quad \text{Equation 22}$$

$$\alpha = \sin^{-1}\left(\frac{\cos i}{\cos \phi}\right) \quad \text{Equation 23}$$

$$\Delta v_{rot} = -\frac{2\pi\, R_{Earth}}{t_{day}} \cos(\phi)\sin(\alpha) \qquad \text{Equation 24}$$

$$\Delta v_{req} = v_{orbit} + \Delta v_{poten} + \Delta v_{rot} + \Delta v_{losses} \qquad \text{Equation 25}$$

$$h_{orb}(\Delta v_{avail}) = \frac{\mu}{\frac{2\mu}{R_{Earth}} - (\Delta v_{avail} - \Delta v_{rot} - \Delta v_{losses})^2} - R_{Earth} \qquad \text{Equation 26}$$

where
h_{orb} = target orbit altitude
R_{Earth} = equatorial radius of Earth (6.378×10^6 m) [2]
μ = gravitational parameter of Earth (3.986×10^{14} $m^3\ s^{-2}$) [2]
α = launch azimuth
ϕ = launch site latitude
i = target orbit inclination
v_{orbit} = burnout velocity; the velocity in orbit you intend to reach
Δv_{poten} = delta-V required by the change in potential energy
Δv_{rot} = delta-V due to the rotation of the Earth
Δv_{losses} = delta-V corresponding to losses incurred by atmospheric drag, steering, and gravity
Δv_{req} = delta-V required to reach the intended orbit
t_{day} = length of the sidereal day in seconds (86164 s) [2]

Using the reference orbit provided by SpaceX (Table 7), I calculated the maximum accessible orbit altitude for a range of payload masses. The results are plotted in Figure 5 along with the rated payload mass advertised by SpaceX. According to my calculations, the reference 500 km orbit inclined to 98.6 degrees is *not* accessible with the rated payload mass of 100 Mg. However, Starship appears able to launch a respectable 79.3 Mg of payload to the reference orbit. As previously noted, several of my calculations are based on conservative estimations, and over successive calculations the conservativeness likely compounds leading to a large range of possible outcomes.

Table 7. SpaceX LEO reference orbit and launch site parameters

Name	Symbol	Value	Unit	Source
Payload mass to orbit – single launch (reference)	m_{ref}	100+	Mg	[7]
Orbit altitude (reference)	h_{ref}	500	km	[7]
Orbit inclination	i	98.9	deg	[7]
Launch site latitude (Boca Chica, TX)	ϕ	26	deg	[31]

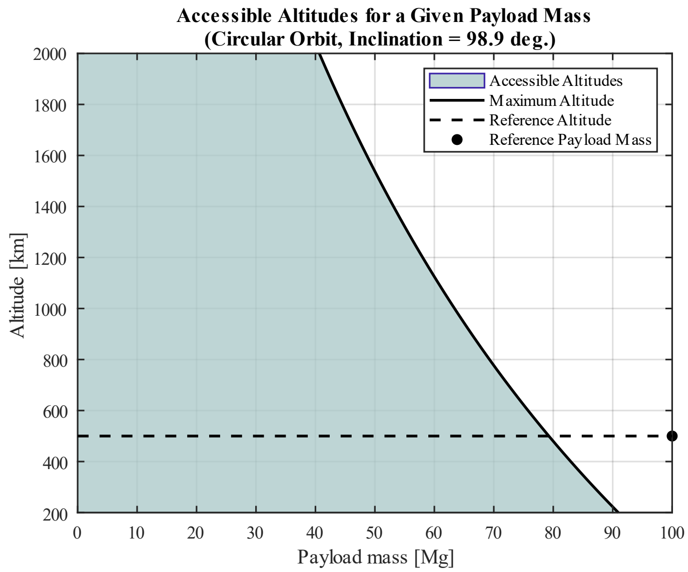

Figure 5. Accessible altitudes for a given payload mass

But the highly inclined, slightly retrograde sun synchronous orbit (SSO) is not an ideal orbit when considering the delta-V costs. To maximize delta-V—and therefore payload mass to orbit—a rocket should ideally launch directly eastwards to take full advantage of Earth's rotation. In other words, Starship not having the capability to launch 100 Mg to the reference SSO, does not preclude it from launching the same payload mass to orbits of other inclinations. Using the same methodology as the previous example, I calculated the maximum accessible orbit altitude for a given payload mass for a range of inclinations. The inclinations start at 26 degrees because inclinations less than the value of the launch latitude yield an imaginary result when launch azimuth is calculated using Equation 23. Looking at the results in Figure 6, it is evident that the delta-V gained from the Earth's rotation has a relatively significant effect on the achievable altitude for a given payload mass. Starship is capable of lofting 100 Mg payloads to 500 km in orbits with inclinations of about 64 degrees or less. It should be noted that the x-axis of Figure 6 begins at 40 Mg (in contrast to Figure 5) to make the individual inclination contour lines more distinct.

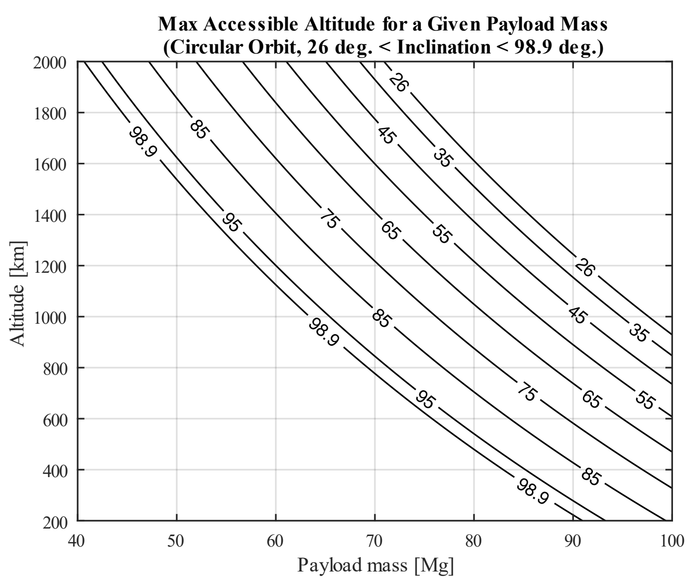

Figure 6. Accessible altitudes for a given payload mass

3. Geosynchronous Transfer Orbit (GTO)

Another popular orbit destination is geosynchronous transfer orbit (GTO). Like it says in the name, a GTO is an orbit for transferring between an initial LEO parking orbit and the final geosynchronous equatorial orbit (GEO), a circular orbit with an altitude of 35,786 km [32]. Since GEO has a period equal to Earth's rotation, GEO is commonly used for communications satellites that require being in a fixed location in the sky relative to Earth. However, reaching GEO is expensive in the delta-V sense. Direct insertion of a payload by a launch provider requires a very capable launch vehicle and is often prohibitively expensive. Military spacecraft might opt for direct to GEO insertion, but commercial spacecraft—making up a majority of satellites in GEO—tend to opt for the GTO route and carry extra fuel onboard for performing the final circularization maneuver at GEO [33].

SpaceX currently advertises 21 Mg of payload to GTO in a single launch. The GTO is specifically referenced as being inclined to 27 degrees, having a perigee altitude of 185 km, and an apogee altitude of 35,786 km (Table 8). The easiest way to calculate the total delta-V required for the destination orbit, two separate events will be considered: launch to a parking orbit with altitude equal to the perigee altitude of the GTO and an instantaneous maneuver placing the Starship into a GTO.

Table 8. SpaceX GTO reference orbit and launch site parameters

Name	Symbol	Value	Unit	Source
Payload mass to orbit – single launch (reference)	m_{ref}	21	Mg	[7]
Orbit perigee (reference)	$h_{ref,peri}$	185	km	[7]
Orbit apogee (reference)	$h_{ref,apo}$	35,786	km	[7]
Orbit inclination	i	27	deg	[7]
Launch site latitude (Boca Chica, TX)	ϕ	26	deg	[31]

First, the GTO perigee velocity ($V_{GTO,peri}$) is calculated in Equation 28, where the semimajor axis (a) is found from Equation 27. The velocity of the parking orbit (V_{park}) is calculated (Equation 29) which is then used to find the delta-V needed to perform the transfer maneuver (ΔV_{GTO}) (Equation 30). Using the same methods as the previous section, the delta-V required to reach the parking orbit (ΔV_{park}) is calculated as well as the delta-V available from Starship and Super Heavy (ΔV_{avail}). Subtracting ΔV_{park} and ΔV_{GTO} from the total amount of delta-V available (ΔV_{avail}) yields a net delta-V that Starship may use for further maneuvers. This value should non-negative if the destination orbit lies within the delta-V capabilities of Starship. However, the value of ΔV_{remain}*is* negative, meaning that, by my calculations, Starship does not possess the capability to launch 21 Mg directly into GTO.

$$a = \frac{h_{ref,peri} + h_{ref,apo}}{2} + \mathrm{R_{Earth}} \quad \text{Equation 27}$$

$$\mathrm{v}_{GTO,peri} = \sqrt{\frac{2\mu}{\mathrm{R_{Earth}}} - \frac{\mu}{a}} \quad \text{Equation 28}$$

$$\mathrm{v}_{park} = \sqrt{\frac{\mu}{\mathrm{R_{Earth}} + \mathrm{h_{park}}}} \quad \text{Equation 29}$$

$$\Delta\mathrm{v}_{GTO} = \left|V_{GTO,peri} - V_{park}\right| \quad \text{Equation 30}$$

$$\Delta\mathrm{v}_{remain} = \Delta\mathrm{v}_{avail} - \Delta\mathrm{v}_{park} - \Delta\mathrm{v}_{GTO} \quad \text{Equation 31}$$

where a = semi-major axis of the orbit
$v_{GTO,peri}$ = perigee velocity of GTO
v_{park} = parking orbit velocity
Δv_{GTO} = delta-V required to change orbits from the parking orbit to GTO
Δv_{remain} = delta-V remaining after reaching the target orbit
Δv_{avail} = delta-V available to the launch system

In fact, I found that Starship is not able to launch *any* amount of payload into GTO (Figure 7). Starship appears able to reach GTO only if its dry mass is reduced by 19 Mg and without carrying any payload mass. Interestingly, during the September 2019 Starship Update presentation the dry mass of Starship was originally listed as 85 Mg in the presentation slides, but Musk verbally indicated the value as an error remarking "I wish it was 85 tonnes!" and correcting the value to "approximately 120 [Mg]" [3]. Using this value, I calculated about 19 Mg of payload mass to GTO (Figure 7), which lines up nicely with the 21 Mg of payload mass stated in the Starship Users Guide—originally published in March 2020 and not revised since. There is possibility—albeit a small one—that the individuals at SpaceX responsible for publishing the Starship Users Guide could have used payload estimates derived from outdated dry mass figures. The document was not published until about six months after the update presentation, but the document could have been in the works well before then and the error just never caught. However, I do not find the idea of SpaceX being unaware of a possible error in what is effectively an advertisement to customers—particularly the payload mass to orbit, an important figure for spacecraft designers. Rather, I would believe it to be more likely that making slightly conservative estimates for multiple subsystems—engine performance, delta-V ascent losses, dry mass figures, propellant reserve masses, etc.—cascades through my calculations leaving me with a rocket that underperforms on paper.

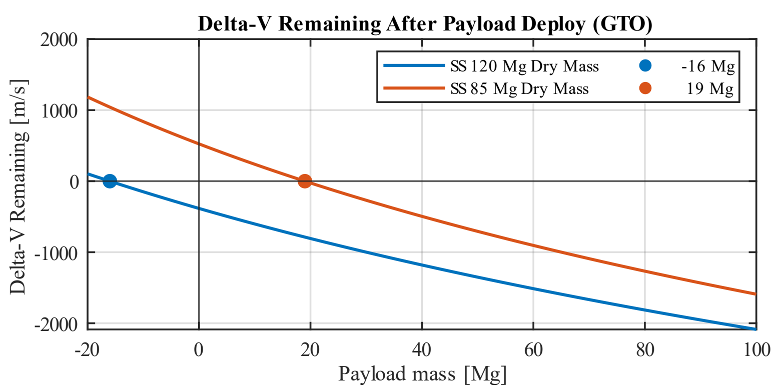

Figure 7. Delta-V remaining after GTO payload deployment

4. Orbital Refueling Mission

Once operational, Starship will have an ability that no other launch vehicle or spacecraft has ever had: the ability to refuel in orbit through a massive transfer of cryogenic propellants. In theory a Starship—nearly out of propellant after its initial launch into orbit—with up to 100 Mg of payload destined for Mars is able to dock with tanker Starships and refuel before finally leaving for the Red Planet [12, 7]. This concept effectively "resets" the classic rocket equation, enabling interplanetary missions with truly massive payloads.

The success of orbit refueling depends somewhat on the amount of propellant that can be delivered to the awaiting spacecraft, and thus, how many refueling missions the spacecraft requires before it has the necessary propellant to perform its primary mission. So how much propellant can Starship deliver to orbit? Well, it depends on the mission architecture SpaceX decides to employ. I see several potential avenues: a normal cargo Starship with no payload, a normal cargo Starship with 100 Mg of propellant as a "payload," a specially made tanker Starship, or a special tanker Starship rendezvousing with the target Starship in a slightly lower parking orbit.

Propellant mass to orbit also depends somewhat on the target orbit altitude and inclination. Obviously, the inclination of the orbit should be equal to the launch latitude, allowing the launch vehicle to take advantage of the Earth's rotation as much as possible, and the altitude should be as low as practical to reduce the delta-V costs as well. However, if the orbit is too low, the more significant drag force compared with higher altitudes will cause the orbit to decay over time. As a counterpoint, Starship will have, particularly after several refuelings, a significant amount of inertia to resist the force of drag, so perhaps the drag at lower altitudes is not as much of a concern. In any case, for this analysis I have somewhat arbitrarily chosen 300 km as the altitude for the parking orbit as Starship waits to be refueled. It seems to strike a balance between too low and too high. As a reference, SpaceX's Starlink satellites are normally deployed into a 380 km circular parking orbit [34].

Not much needs to be said about the first architecture option. By launching to a particular orbit with no payload, a certain amount of delta-V, and thus propellant, goes unused and is available for transfer to the target vehicle. This excess delta-V ($\Delta \mathrm{v}_{\mathrm{excess}}^{\mathrm{SS}}$), from Equation 32, is subtracted from the delta-V available in the Starship vehicle ($\Delta \mathrm{v}^{\mathrm{SS}}$) to find the amount of delta-V that Starship will expend to reach its target orbit ($\Delta \mathrm{v}_{\mathrm{req}}^{\mathrm{SS}}$)(Equation 33). Knowing the initial mass of Starship (m_f), a form of the rocket equation can be used to find the mass of Starship immediately after the maneuver (Equation 34). The excess propellant ($m_{excess\ prop}$), unused propellant available for transfer to the target vehicle, is a simple matter of subtraction (Equation 35).

$$\Delta \mathrm{v}_{\mathrm{excess}}^{\mathrm{SS}} = \Delta \mathrm{v}_{\mathrm{avail}} - \Delta \mathrm{v}_{req} \qquad \text{Equation 32}$$

$$\Delta \mathrm{v}_{\mathrm{req}}^{\mathrm{SS}} = \Delta \mathrm{v}^{\mathrm{SS}} - \Delta \mathrm{v}_{\mathrm{excess}}^{\mathrm{SS}} \qquad \text{Equation 33}$$

$$m_f = \mathrm{m_i} e^{\left(\frac{-\Delta \mathrm{v}_{\mathrm{req}}^{\mathrm{SS}}}{c}\right)} \qquad \text{Equation 34}$$

$$m_{excess\ prop} = m_f - m_{dry} - \mathrm{m_{headerProp}} - \mathrm{m_{payload}} \qquad \text{Equation 35}$$

where
$\Delta \mathrm{V}_{\mathrm{excess}}^{\mathrm{SS}}$ = delta-V available to Starship after reaching the target orbit
$\Delta \mathrm{V}_{\mathrm{req}}^{\mathrm{SS}}$ = delta-V required for Starship to reach the target orbit
m_f = mass of Starship after entering the target orbit
$\mathrm{m_i}$ = mass of Starship before performing maneuver
c = effective exhaust velocity
$m_{excess\ prop}$ = mass of propellant remaining after successfully entering the target orbit

Alternatively, the payload itself could be propellant. However, I do not believe, that SpaceX would engineer a Starship with extra propellant tanks in the payload bay solely for propellant transfer. Rather, it would be simpler, and likely cheaper, to increase the tank height to accommodate the extra 100 Mg of propellant. As it turns out, part of the nose cone is constructed of a nine-meter-tall cylinder section. A portion of this section can be converted into tankage by simply adjusting the location of the propellant tank bulkheads along the length of the ship allowing the propellant tanks to increase in height without increasing the height or dry mass of Starship as a whole. Determining the extra height of this tankage is a simple matter of density, volume, and geometry. Ultimately the extra propellant tank height needed to accommodate the 100 Mg increase in propellant mass ($\hbar_{additional}$) is a mere 0.531 meters (Table 9).

Table 9. Parameters for tankage stretch due to propellant payload mass

Parameter	Symbol	Value	Units	Source
Propellant payload mass	$m_{prop\ payload}$	100	Mg	[7]
LOX density (at 66 K)	ρ_{LOX}	1255.4	kg m^{-3}	[35]
CH_4 density (at 102 K)	ρ_{CH4}	435.5	kg m^{-3}	[35]
LOX mass	m_{LOX}	23.41	Mg	Equation 36
CH_4 mass	m_{CH4}	6.60	Mg	Equation 37
LOX volume	$\mathbb{V}_{LOX}$	18.65	m^3	Equation 38
CH_4 volume	$\mathbb{V}_{CH4}$	15.14	m^3	Equation 38
LOX height	$\hbar_{LOX}$	0.293	m	Equation 39
CH_4 height	$\hbar_{CH4}$	0.238	m	Equation 39
Total extra height	$\hbar_{additional}$	0.531	m	Equation 40

$$m_{LOX} = \frac{O/F}{O/F + 1} m_{prop\ payload} \quad \text{Equation 36}$$

$$m_{CH4} = \frac{1}{O/F + 1} m_{prop\ payload} \quad \text{Equation 37}$$

$$\mathbb{V}_i = \frac{m_i}{\rho_i} \quad \text{Equation 38}$$

$$\hbar_i = \frac{\mathbb{V}_i}{\pi (D^{SS})^2} \quad \text{Equation 39}$$

$$\hbar_{additional} = \hbar_{LOX} + \hbar_{CH4} \quad \text{Equation 40}$$

However, if SpaceX begins down this path of creating a custom tanker variant of Starship, it is not unlikely that other design alterations might be made. For one, the previously mentioned nine-meter-tall cylinder section of the nose cone could potentially be completely removed. The nose cone is primarily for reducing aerodynamic drag during ascent, and the removal of the cylindrical section should not appreciably alter performance. Taking the density of stainless steel to be 8 $\mathrm{Mg/m^3}$ and the average thickness of 4 mm, this change alone would result in a reduction of 6540 Mg [36]. Further, the payload bay door would not be needed. Mass estimates for the hardware associated with the door as well as any structural support needed for reinforcing the area around the large hole do not currently exist (not publicly anyway). But I do not think it would be unreasonable to conservatively estimate that about 3460 Mg could be removed allowing the total dry mass reduction to nicely add up to 10 Mg.

Lastly, simply choosing a lower altitude for the parking orbit could significantly increase propellant mass delivered to the target vehicle. Again, I have somewhat arbitrarily chosen a lower altitude of 250 km for the parking orbit. The aerodynamic drag at this altitude will no doubt be higher, but perhaps with the relatively short loiter time and relatively high mass inertia there might not be much of an problem. Of course, both the original and lower altitudes are arbitrarily chosen, but the point of this exercise is to show what is possible for a tanker somewhere in LEO as well as the difference when compared with an orbit slightly lower in altitude.

Neatly summarized in Table 10 and plotted in Figure 8 below, I performed the calculations using each of the four methods above. The largest increase in propellant delivery capacity (11 Mg) is a result of the extra 100 Mg of propellant onboard. Nine additional megagrams are added by reducing the dry mass of the Starship with extended tanks. Lastly, only two extra megagrams of propellant were available for transfer when the transfer occurred at an orbit 50 km lower in altitude. For SpaceX, getting a system into an operational state is key to success as evident by the Falcon 9 program which experienced a myriad of upgrades and improvements over the years. Starship is no exception, and I believe it would be optimal for SpaceX to begin refueling flights with a simple empty cargo Starship as this route minimizes development costs and allows the company to begin testing large volume cryogenic propellant transfers as early in the development cycle as possible. Later, the company may decide to build a dedicated tanker variant of Starship to have an additional 20 Mg or more for propellant transfers. It is likely that Starship can launch more than the 100 Mg additional propellant I used in my calculations, but it remains unknown what exactly the upper limit might be. The limited is primarily due to the thrust-to-weight ratio of the vehicle. If Raptor engines are improved in the same manner that the Merlin engines of the Falcon 9 have been improved over the years, Starship could see significant gains in propellant mass to orbit. Dry mass reductions beyond my 10 Mg estimation would further increase propellant mass to orbit capability.

Table 10. Propellant refueling mission parameters

Method	Description	Dry mass (Mg)	Altitude (km)	Propellant Payload Mass (Mg)	Propellant Mass Delivered (Mg)
Method 1	Empty normal Starship	120	300	0	112
Method 2	Extended tanks	120	300	100	123
Method 3	Extended tanks, lightened ship	110	300	100	132
Method 4	Extended tanks, lightened ship, lower altitude	110	250	100	134

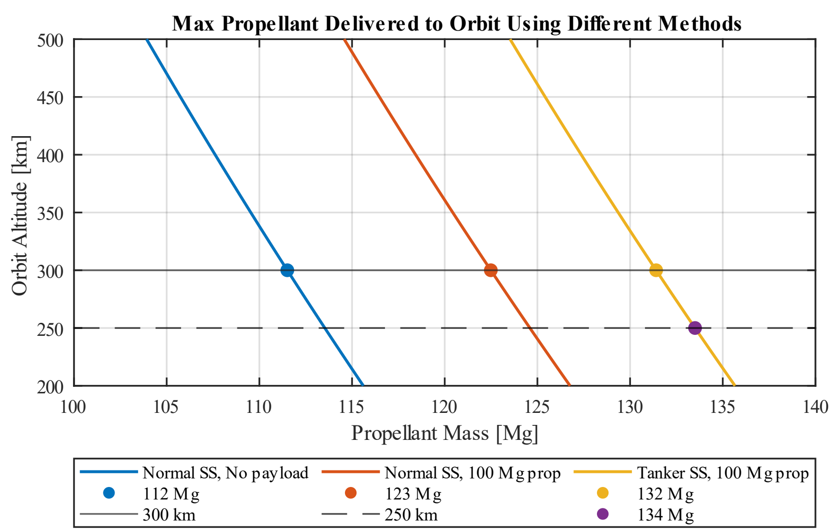

Figure 8. Max propellant delivered per refueling mission

D. Orbit Refueling

1. High Energy Trajectory: Direct Insertion

In-orbit refueling is a bit of a double-edged sword for the Starship program. Having the technology for transferring hundreds of megagrams of cryogenic propellant between vehicles in orbit will be an absolute game changer and open up mission profiles previously thought impossible. On the other hand, its arguable that the success of refueling in orbit is critical to the success of the Starship program as a whole. Like the Space Shuttle, Starship carries around extra dry mass in the form of a heat shield, landing gear/legs, and other equipment for reusability. If Starship is going to avoid the fate of the Shuttle as an expensive U-Haul® stuck at low Earth orbit, then orbit refueling *must* work.

But suppose it does work, technologically speaking. How would it help? What missions would it suddenly make available? Well, it is actually a relatively straightforward calculation. First, the total mass of Starship (m_0) is calculated at whatever propellant level ($m_{mainProp}$) is being investigated (Equation 41). Then, the total mass of Starship (m_1) is calculated absent any of the main propellant (Equation 42). These two different values, representing the change in propellant mass before and after an orbit maneuver, are used in Equation 43 to find the delta-V (Δv) for that particular maneuver. The delta-V can also be turned into the characteristic energy (C_3), or hyperbolic excess velocity, which is an amount of specific energy needed for a maneuver beyond the amount of specific energy needed to just barely escape the gravitational influence of the Earth [37]. This calculation (Equation 44) takes into account the current position of the spacecraft (r) since delta-V is referenced to the location of the spacecraft as opposed to the characteristic energy which is referenced to whatever specific energy or delta-V it takes for a spacecraft to escape the gravity of Earth. Moreover, this property is actually quite useful to interplanetary mission planners since one does not need to know what parking orbit the spacecraft will stop at or if it will even stop at a parking orbit at all. The number of refueling flights needed (n_{tanker}) depends entirely on the amount of main propellant needed for the maneuver, calculated by the ceiling of the propellant needed divided by the amount of propellant that Starship can loft into orbit ($\mathrm{m_{propPayload}}$). From the results of the previous section, I have calculated that an empty Starship can bring somewhere between approximately 110 Mg and 130 Mg to a 300 km altitude circular orbit. I will use both figures in my calculations to show upper and lower bounds.

$$\mathrm{m_0 = m_{mainProp} + m_{headerProp} + m_{dry} + m_{payload}} \qquad \text{Equation 41}$$

$$\mathrm{m_1 = m_{headerProp} + m_{dry} + m_{payload} = m_0 - m_{mainProp}} \qquad \text{Equation 42}$$

$$\Delta \mathrm{v} = -c \ln\left(\frac{\mathrm{m_1}}{\mathrm{m_0}}\right) \qquad \text{Equation 43}$$

$$\mathrm{C_3} = \left(\Delta \mathrm{v} + \sqrt{\frac{\mu}{\mathrm{r}}}\right)^2 - \frac{2\mu}{\mathrm{r}} \qquad \text{Equation 44}$$

$$n_{tanker} = \left\lceil \frac{\mathrm{m_{mainProp}}}{\mathrm{m_{propPayload}}} \right\rceil \qquad \text{Equation 45}$$

where
$\mathrm{m_0}$ = mass of Starship before the maneuver
$\mathrm{m_1}$ = mass of Starship after the maneuver
$\Delta\mathrm{v}$ = delta-V cost of the maneuver
$\mathrm{C_3}$ = characteristic energy
r = position in orbit
n_{tanker} = number of required refueling missions (rounded up)

To get the "bigger picture" of the effects of orbital refueling, I used MATLAB to perform the above calculations for a range of payload masses, delta-V requirements, and amount of main propellant. The results are plotted (Figure 9) as a filled contour plot with a color bar for the number of flights required (Equation 45). Delta-V is plotted along the left vertical axis, but the corresponding characteristic energy is plotted along the right vertical axis for convenience or conversion. Each of the colored bands on the plot represents the space of compatible payload masses and trajectory energies for a given refuel flight number. For example, in figure below, there is a yellow horizontal line indicating the departure energy to Mars in the 2022 launch window is C_3 = 13.79 km^2/s^2 [38]. For payload mass projected onto this yellow line, there is a corresponding number of propellant refuel stages required send the Starship on this particular trajectory. In this specific example, using the chart corresponding to 110 Mg of propellant delivered each flight, payload masses ranging from 0 Mg to about 20 Mg require Starship to be delivered propellant three times, payload masses from about 20 to 75 Mg require four refueling flights, and lastly, payloads massing 75 Mg up to 100 Mg will require five tanker flights. In the more optimistic case with 130 Mg being delivered each flight, three tanker flights are required for payload masses between 0 Mg and 60 Mg, and four flights are needed if payload mass ranges from 60 Mg to 100 Mg. These results are close to the advertised capabilities of a refueled Starship. Elon Musk has said that Starship could get to Mars with about four refueling flights and that using a well optimized tanker vehicle, a Starship could be completely refueled in five or six trips [39].

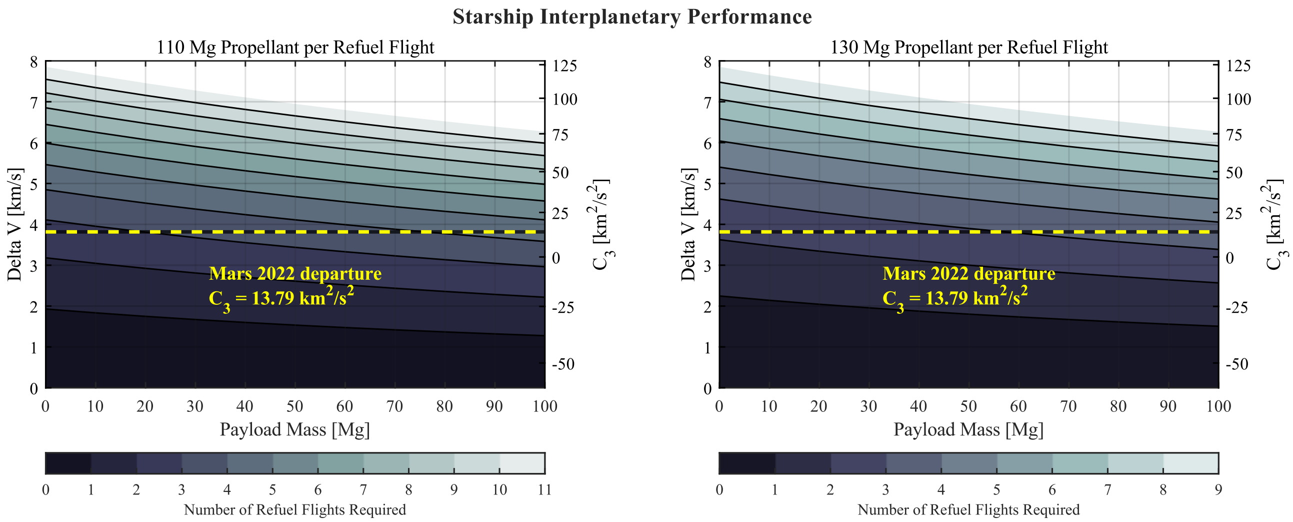

Figure 9. Starship interplanetary performance in terms of payload mass, delta-V/characteristic energy, and number of refuel flights required

2. High Energy Trajectory: "Burn and Return"

The previous trajectory type would be used if Starship were required to arrive at the destination. For example, if the mission profile is landing cargo or crew on the surface of Mars, it should be quite clear that Starship is required for safely touching down on the surface. But this type of trajectory is not optimal for all mission profiles. Spacecraft on flyby missions and other space probes like the Voyagers, New Horizons, OSIRIS-Rex, have their own control systems and propellant. As such, these spacecraft would not directly benefit from Starship after deployment of the spacecraft. In the best case, Starship would be on a trajectory that would eventually encounter Earth after several months where it can land, be inspected, and fly more missions. In the worst case, the Starship vehicle is placed in a trajectory such that there is no opportunity for landing back on Earth in any of the future orbit propagations. During the long period between launch and landing in the former scenario, Starship is not available for flying additional missions to make SpaceX money; in the latter scenario, Starship will never be available to fly another mission.

Therefore, it is in the best interest of SpaceX to return its Starship vehicles to Earth as quickly as possible. As a potential solution to this problem, I would like to explore a mission where Starship refuels in LEO, performs a maneuver placing itself on the target trajectory, releases the spacecraft, rotates 180 degrees, and performs a final maneuver using the remaining main propellant onboard to place itself on a suitable trajectory to land back on Earth. I call it the "burn and return." Part of the difficulty here is determining just how much propellant to use for the trajectory insertion burn ($m_{propUsed}$) and how much needs to be saved for the return burn ($m_{propRemain}$). For this analysis, I will not be getting into the orbital mechanics of this problem; I will keep it simple and assume that the delta-V for the first maneuver ($\Delta v_{outbound}$) must be equal—and opposite in direction—to the second maneuver (Δv_{return}) allowing the relative velocities to cancel. In this scenario the maneuvers and payload release happen instantaneously resulting in Starship remaining in its original LEO parking orbit after all is said and done. It is typical in orbital mechanics problems to assume instantaneous maneuvers, particularly for first-order estimations like this one; planets and orbits are quite large in comparison to the amount of distance a rocket can travel in a short amount of time.

To begin, the masses of Starship at different stages are considered. The following should be true by intuition: $m_0 > m_1 > m_2 > m_3$, where m_0 is the mass of Starship in LEO before the maneuver, m_1 is the mass of Starship immediately after the maneuver but before payload deployment, m_2 is the mass of Starship immediately after the payload deployment but before the return maneuver, and lastly, m_3 is the mass of Starship immediately after then return maneuver. The exact values of the mass of Starship at all four stages are calculated by Equation 47, Equation 48, Equation 49, and Equation 50.

$$\mathrm{m_{mainProp}} = \mathrm{m_{propUsed}} + \mathrm{m_{propRemain}} \qquad \text{Equation 46}$$

$$\mathrm{m_0} = \mathrm{m_{mainProp}} + \mathrm{m_{headerProp}} + \mathrm{m_{dry}} + \mathrm{m_{payload}} \qquad \text{Equation 47}$$

$$\mathrm{m_1} = \mathrm{m_0} - \mathrm{m_{propUsed}} \qquad \text{Equation 48}$$

$$\mathrm{m_2} = \mathrm{m_1} - \mathrm{m_{payload}} \qquad \text{Equation 49}$$

$$\mathrm{m_3} = \mathrm{m_2} - \mathrm{m_{propRemain}} \qquad \text{Equation 50}$$

$$\Delta\mathrm{v_{outbound}} = -c \ln\left(\frac{\mathrm{m_1}}{\mathrm{m_0}}\right) \qquad \text{Equation 51}$$

$$\Delta\mathrm{v_{return}} = -c \ln\left(\frac{\mathrm{m_3}}{\mathrm{m_2}}\right) \qquad \text{Equation 52}$$

I used MATLAB to solve for $\Delta\mathrm{v_{outbound}}$ and $\Delta\mathrm{v_{return}}$ at all possible values of $\mathrm{m_{propUsed}}$. Plotting both ranges of delta-Vs in terms of $\mathrm{m_{propUsed}}$ (Figure 10), it becomes clear that there must be only one solution to $\Delta\mathrm{v_{outbound}} = \Delta\mathrm{v_{return}}$ and what that value of delta-V is. So far only a single value of delta-V is returned corresponding to 50 Mg of payload on the same Mars 2022 launch trajectory as above. I calculated the data for Figure 11 in the same manor that I calculated the data for Figure 9. This portion of the analysis has a few assumptions baked in. First, I am assuming that the most fuel-efficient method to perform a "burn and return" type mission is to fuel up completely in LEO and perform all of the maneuvers there. However, it could also be the case that refueling stages could take place in highly elliptical orbits similar to the method SpaceX proposed several years ago for their lunar surface missions [40]. Or the optimal method could be something completely different.

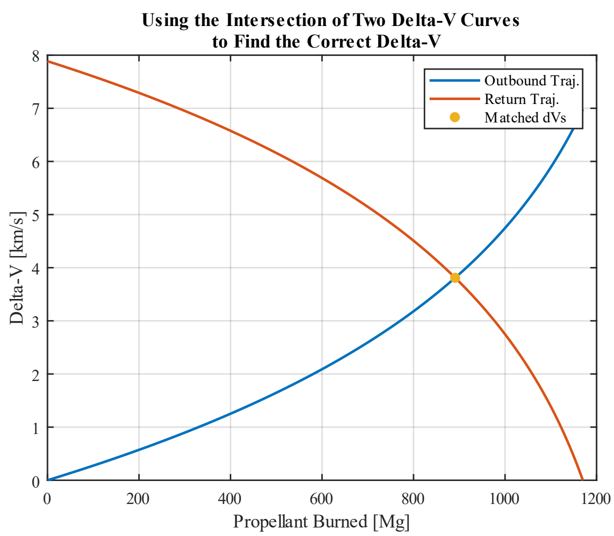

Figure 10. Using $\Delta\mathbf{V_{outbound}}$ and $\Delta\mathbf{V_{return}}$ as a function of $\mathbf{m_{propUsed}}$ to find $\Delta\mathbf{V_{outbound}} = \Delta\mathbf{V_{return}}$

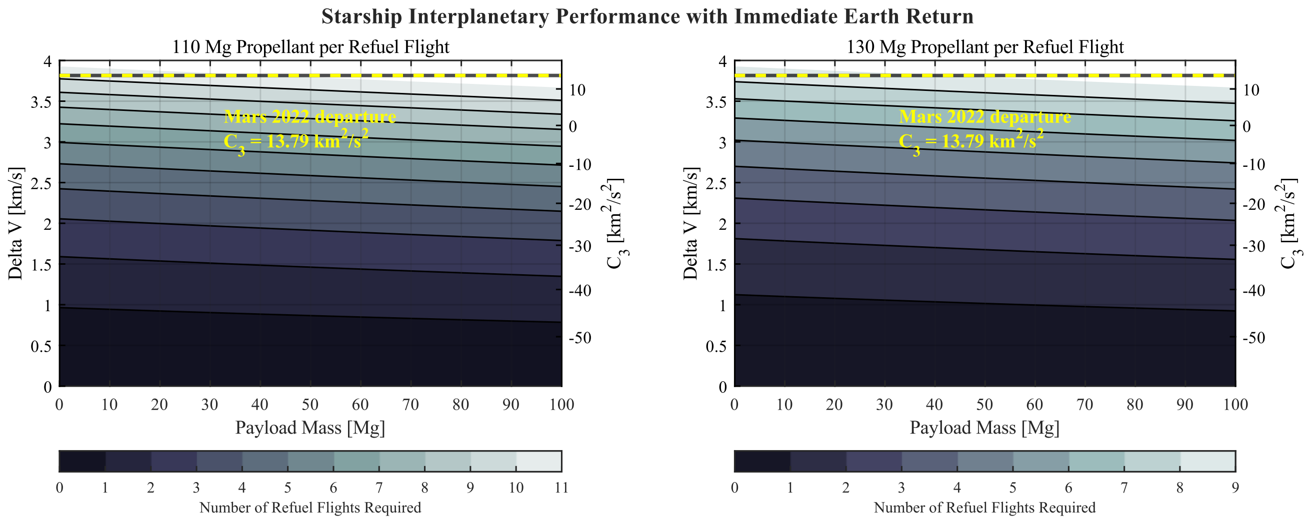

Figure 11. Starship interplanetary performance with an immediate return to Earth in terms of payload mass, delta-V/characteristic energy, and number of refuel flights required

Launching a mission to Mars—or any other similar trajectory—using the "burn and return" method will always be at the edge of Starship's capabilities, requiring almost completely full propellant tanks due to diminishing returns with orbital refueling. Whether using the "burn and return" method to prevent needing a kick-stage is worth launching nearly a dozen or so refueling missions remains to be seen, however, I suspect that the cost of even a large solid rocket kick-stage will be less expensive than about a dozen Starship and Super Heavy launches, even if the only cost incurred is for propellant. Nonetheless, it was an interesting view into how the need to save a large amount of propellent for the return burn really affected the performance of the vehicle.

Alternatively, SpaceX may decide that it would be beneficial to construct a lightweight, inexpensive, disposable Starship solely for high-energy interplanetary missions. Dry mass could be reduced significantly by removing, the three sea level engines, header tanks, landing legs, heat shield, flaps, and any other landing hardware. Rockets using liquid propellants tend to have higher thrust and specific impulse than their solid propellant counterparts, a fully fueled Starship would have far more delta-V than any kick-stage that could fit in the payload bay, and any amount of payload mass or volume saved by opting for a disposable Starship over including a kick-stage could be budgeted back to the spacecraft. Though, these options are not mutually exclusive, and a truly high-energy mission could possibly employ both a disposable Starship and an additional kick-stage. I long for the day I get to see a mission like this one actually take place.

E. Landing

1. Background

SpaceX's Starship program is being developed with full and rapid reusability in mind. Unlike the Falcon 9 program, SpaceX will attempt to recover the second stage vehicle from orbit. Safely landing Starship is fundamentally more difficult than simply landing a suborbital first stage booster; when re-entering Earth's atmosphere, the heating rate is proportional to the velocity of the spacecraft cubed which is quite significant for vehicles traveling at orbital velocities [41]. When the Space Shuttle re-entered the atmosphere, it flew more like a plane; the wings generated lift, and the rudder and elevons operated similarly to conventional aircraft control surfaces [42]. Starship, by comparison, is being developed with a novel re-entry method. The reusable spacecraft will enter the atmosphere at a much higher, about 70-degrees, angle of attack, and its four control surfaces will create far more drag than lift [43]. In fact, Starship will be steered through re-entry using differential drag depending on the angle of the flap in each corner. Once Starship horizontal velocity of Starship has decreased enough, it will begin to fall straight down to Earth belly first. This maneuver has colloquially become known as the "belly-flop maneuver" or the "skydive maneuver" [3]. As it falls Starship decelerates until it reaches its terminal velocity. Before hitting the ground, the three sea level Raptor engines ignite to reorient the spacecraft with its nose facing the local vertical. The flight computer then shuts off one or two of the engines allowing for a safe, soft touchdown. Or at least that is how it is supposed to happen. Until recently, SpaceX did not know for certain whether Starship would have adequate control with its flaps for the vertical descent. A total of five suborbital flights have taken place so far to test the controlled descent; the first four vehicles, SN8 through SN11 all experienced an anomaly during the "flip" maneuver and were lost, but on 5 May 2021, Starship SN15 finally made a successful controlled landing [16].

It is this controlled vertical descent that I wish to investigate. Specifically, I want to see how the different throttle settings on the Raptor engines affect the altitude at which the engines light and how much propellant gets used. For this analysis, I will assume a constant thrust for the duration of the landing burn.

2. Horizontal Descent

First, it is necessary to calculate the terminal velocity of Starship as it descends. For this calculation I need the surface area of Starship projected into the airflow. Most of the body of the vehicle is a simple cylinder, but other areas like the nose and flaps are more complex shapes where the projected area is no longer a simple calculation. Instead of approximating the entire vehicle as a cylinder, which would overestimate the area around the nose, I created a 3D model of the Starship vehicle using SolidWorks. The exact dimensions for construct the model are sourced from two schematics drawn by Rafael Adamy [44]. The schematics (Figure 15 and Figure 16) can be found in the Appendix. From the 3D model, the projected area of just the main cylindrical body, including the nose, has a projected area of 446.6 m^2. The projected areas for the flaps are more difficult since the area measurement will depend on the deployment angle of the flap. To simplify the analysis, I chose a 45-degree angle of attack. This position was selected as the neutral position of the flaps during the horizontal descent using video from official Starship test flight webcasts. A video frame of the webcast is available (Figure 17) in the Appendix for reference [16]. Using SolidWorks, I determined the individual fore and aft flap projected areas are 16.1 m^2 and 33.4 m^2, respectively. Not knowing the Reynolds number (Re), I initially used a drag coefficient (C_D) of 1.2 for all surfaces to get a first-pass terminal velocity ($\vec{v}_{terminal}$) of $-60\ \mathrm{m/s}$ (Equation 53). This value is used to calculate the Reynolds number (Equation 54), where the characteristic length (L) is equal to the diameter of Starship ($D^{SS} = 9\ m$). With a Reynolds number of 3.7×10^7 , the drag coefficients of the Starship body and flaps are 0.6 and 1.28, respectively, resulting in a final terminal velocity of $-78\ \mathrm{m/s}$ (Equation 53) [45, 46].

$$\vec{v}_{terminal} = -\sqrt{\frac{2\, m_0\, g}{\sum \left(C_{D_i}\, A_{ref_i} \right) \rho}} \qquad \text{Equation 53}$$

$$\mathrm{Re} = \frac{\rho v_{terminal} L}{\mathcal{M}} \qquad \text{Equation 54}$$

where
- $\vec{v}_{terminal}$ = terminal velocity vector
- g = acceleration due to Earth's gravity at sea level (9.80665 m/s^2) [1]
- C_D = coefficient of drag
- A_{ref} = projected area of a surface
- ρ = air density at standard temperature and pressure (1.225 kg/m^3) [1]
- L = characteristic length, in this case the diameter of Starship D^{SS}
- Re = Reynolds number
- $\mathcal{M}$ = dynamic viscosity of the air at sea level (1.789×10^{-5} kg/m/s) [1]

3. Landing

A wide range of engine throttle positions is necessary for performing a successful vertical powered landing. For most current (i.e., expendable) launch vehicles, this feature is not a requirement, and only minimal throttle adjustments are necessary. The Merlin engines of the Falcon 9 allow it to make a soft touchdown back on Earth thanks to a deep throttle capability enabled by its pintle injectors [21]. However, even with just one engine firing for the landing burn, the Merlin engine has too much thrust at its lowest throttle setting, and the landing rocket, now with no payload and almost no fuel remaining, has a thrust-to-weight ratio (TWR) greater than one [4]. A TWR greater than one indicates that the vehicle does not possess the ability to hover; as soon as the velocity vector has been completely negated, the engine must shut off—hopefully not too far above or below the landing pad—lest the rocket begin to fly back up into the air. Starship, on the other hand, will be able to hover due to the higher inert mass fraction and the greater throttle range of the Raptor engines [47]. This ability, while not inherently good from a delta-V perspective where gravity losses must be reduced, allows the system to perform safe landings on unimproved surfaces and avoid potential hazards which will be necessary for landings on the Moon and Mars. Another benefit is to reduce jerk—a rapid change in acceleration—when Starship is landing with humans and sensitive payloads. Figure 12 shows the thrust output for the three sea level Raptor engines on Starship depending on the number of engines firing and the particular throttle setting. There is a continuous range of thrusts from less than 1000 kN at the low end to over 6000 kN of thrust with three engines at maximum throttle. However, the low end of the throttle range should be avoided if possible as a higher risk of engine flameout exists [47].

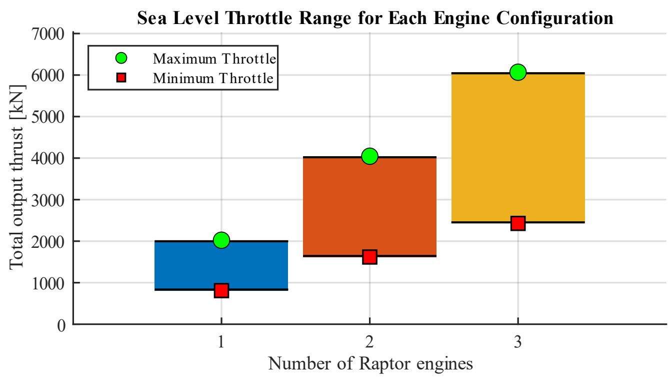

Figure 12. Throttle range for different engine configurations

Assuming Starship flips from horizontal to vertical and begins its landing burn instantaneously, simple kinematic and dynamic equations can be used to model the maneuver. First, it will be assumed that the vehicle only moves in the vertical dimension, the drag experienced by Starship in the vertical orientation will be negligible, and the landing burn is performed under constant thrust ($\vec{F}_{T\,total}$) for a given throttle ($\mathfrak{t}$) input. In reality, the onboard flight computer will be constantly varying the thrust to ensure the vehicle remains oriented correctly, acceleration stays within accepted limits as vehicle mass decreases, and the vehicle does not come to a complete stop too far above or below the ground. Depending on the throttle setting and number of engines participating in the landing burn (n_{eng}), Starship will have a certain amount of time before the vehicle runs out of propellant (t_{empty}) (Equation 56) as dictated by the overall mass flow rate exiting the engines ($\dot{m}_{total}$) (Equation 55). Starship will have a certain mass before the landing burn (m_0), and for this analysis where there is no returned payload mass, the initial mass is 150 Mg (Equation 57).

$$\dot{m}_{total} = n_{eng}\,\mathfrak{t}\,\dot{m} \quad \text{Equation 55}$$

$$t_{empty} = \frac{m_{headerProp}}{\dot{m}_{total}} \quad \text{Equation 56}$$

$$m_0 = m_{headerProp} + m_{dry} + m_{payload} \quad \text{Equation 57}$$

where $\dot{m}_{total}$ = mass flow rate of propellant leaving Starship
n_{eng} = number of active engines
$\mathfrak{t}$ = engine throttle position (40% to 100%) [48]
$\dot{m}$ = mass flow rate of propellant through a single Raptor engine
t_{empty} = time when Starship runs out of propellant

Now, the system can be modeled as a set of three differential equations (Equation 60, Equation 61, and Equation 62), where $\vec{F}_{T\,total}$ is the thrust force (Equation 58) and $\vec{F}_g$ is the force due to gravity (Equation 59). The set of differential equations are ultimately functions of the number of engines actively generating thrust (n_{eng}) and the throttle setting of the engines ($\mathfrak{t}$). I numerically solved the differential equations using the "ode45" function in MATLAB, which employs a version of the Runge–Kutta method, for a range of input engine counts and throttle settings [49]. Specifically, the number of sea level Raptor engines ranged from one to three, and the throttle setting ranged from the minimum of about 40% up to 100% throttle. The complete set of results is plotted in Figure 13.

$$\vec{F}_{T_{total}} = n_{eng} \, \mathbb{t} \, F_T \qquad \text{Equation 58}$$

$$\vec{F}_g = -mg \qquad \text{Equation 59}$$

$$\frac{d\vec{h}}{dt} = \vec{v} \qquad \text{Equation 60}$$

$$\frac{d\vec{v}}{dt} = \frac{\vec{F}_{T_{total}} + \vec{F}_g}{m} \qquad \text{Equation 61}$$

$$\frac{dm}{dt} = -\dot{m}_{total} \qquad \text{Equation 62}$$

where $\vec{F}_{T_{total}}$ = total thrust force vector
$\vec{F}_g$ = gravity force vector
$\vec{h}$ = altitude vector
$\vec{v}$ = velocity vector
m = mass of Starship

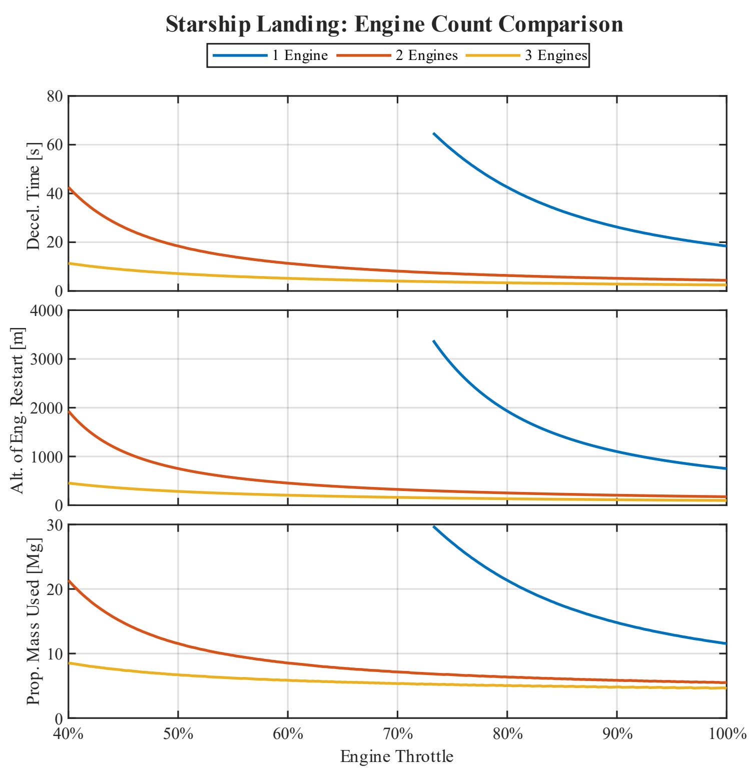

Figure 13. Comparison of engine counts on Starship landing performance

Clearly, for the best performance, all three Raptors should be used at maximum throttle. This method would vastly reduce the deceleration time as well as the altitude at which engine startup occurs thus reducing the delta-V losses due to gravity and reducing the amount of propellant used in the maneuver. Performing the landing burn this way results in a massive 25 Mg of unused landing propellant. Since landing propellant is effectively "dry mass" for the concern of a rocket launch, each megagram of propellant not dedicated for the landing is a megagram that could be used for payload mass or extra propellant for high delta-V missions.

However, many glaring problems exist with this method. First, the acceleration—or more colloquially: deceleration—and force experienced by the vehicle must be limited based on structural considerations. If the flight is crewed, acceleration is limited even more so; for Shuttle flights, acceleration peaked at three times Earth's gravity, but uncrewed vehicles regularly exceed this artificial limit [50]. Second, with three engines firing at maximum throttle, no backup engine exists to use if one fails or underperforms and waiting until the minimum altitude leaves no room for error. Third, this method reintroduces the problems of the Falcon 9 landing as mentioned above.

Using two engines seems like a good middle ground, and this idea is more or less used with Starship landings. Starship will first light all three of its sea level Raptor engines, followed by immediate shutdown of one engine after the flight computer determines the other two are performing nominally [51]. After reducing most of its velocity with two engines, the second engine will be shut down and Starship will land on just one engine [16]. In the first fully successful landing of a Starship prototype on May 5, 2021, Starship SN 15 only lit two of its engines and used both until touchdown after its 15 second powered descent [16]. It is unknown to anyone outside of SpaceX why exactly the flight computer deviated from its nominal instructions, but this anomaly might help me at least partially validate my model.

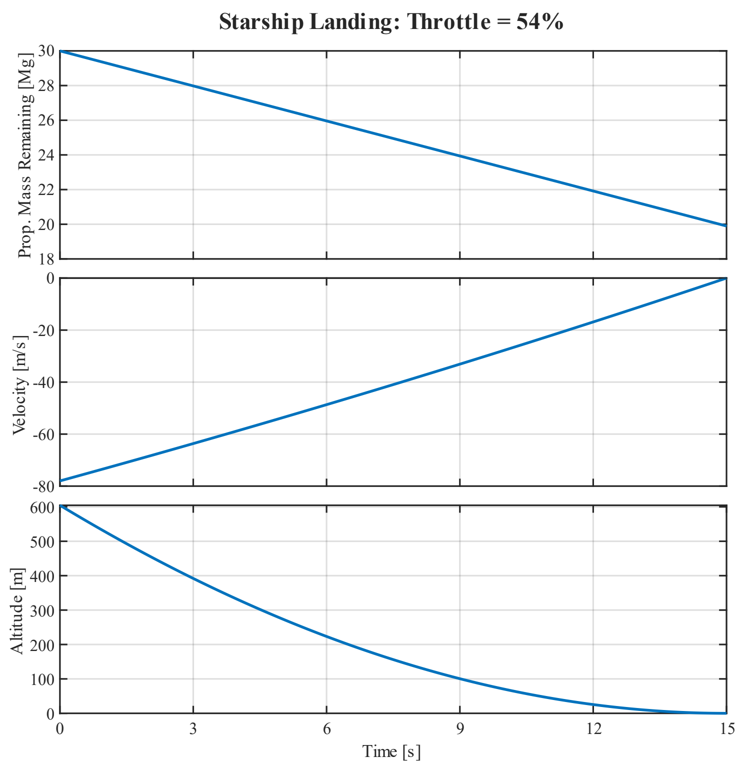

Figure 14. Starship landing performance using two engines at 54% constant throttle

Figure 14 shows the results of my model as calculated above for a constant engine throttle of 54% for two engines. This throttle position corresponds to a deceleration time of about 15 seconds. Under this landing profile, Starship begins its landing burn at an altitude of approximately 600 meters which is close to but slightly higher than the reported altitude of approximately 500 meters [52]. Only about a third of the reserve propellant is used by the landing maneuver, though substantially more could be used with a significant return payload mass.

IV. Conclusion

The only major discrepancies I uncovered with my analysis involved the payload to orbit figures advertised by SpaceX for SSO and GTO missions. The SSO discrepancy could be explained by some of my overly conservative parameter estimates. Though the GTO discrepancy is more difficult to explain, one possible explanation is simply a possible outdated payload estimate calculated from old dry mass figures. As I noted before, using the outdated value for the dry mass of Starship results in a payload mass to GTO that is suspiciously close to the advertised figure. However, this result could be purely coincidental. Otherwise, my results as presented faithfully represent the capabilities of Starship on a higher level.

Multiple paths forward exist to do further research on this system and understand Starship on a deeper level. For one, I would like to explore an actual launch trajectory of Super Heavy and Starship instead of just getting a simple delta-V approximation. This analysis would likely be some type of differential equation boundary value problem. Further, I would like to employ some type of trajectory optimization technique, such as the trapezoidal colocation or direct colocation methods, optimizing for say, a maximum specific energy at stage separation while minimizing the propellant required for first stage landing. I would be interested in seeing how these trajectories compare to current expendable launch vehicles.

Another idea to explore is modeling Starship's high angle of attack hypersonic reentry regime using the flap differential drag to navigate the vehicle to the target landing zone. This topic, while interesting, is outside my area of expertise, however.

SpaceX's Starship program is no doubt tackling some of the most difficult problems within today's launch vehicle industry and leading the charge with aggressive innovation. The Starship vehicle will be one of the most complex vehicles to fly in recent years—on par with the Space Shuttle. The Raptor engines used by Starship have immense power for their size as well as good efficiency allowing the vehicle to loft ridiculous amounts of payload into orbit in a single launch. Any shortcomings Starship may face for high energy missions are more than made up for with orbital refueling. And to top it off, the whole second stage lands back on Earth in a whole new way. From my perspective, the technology is finally here, and the Starship program is feasible. If Starship can live up to its promise of being an ultra-low-cost implementation of reusability—which still remains to be seen—, I think SpaceX with its Starship program will forever change the landscape of the space launch industry.

Appendix

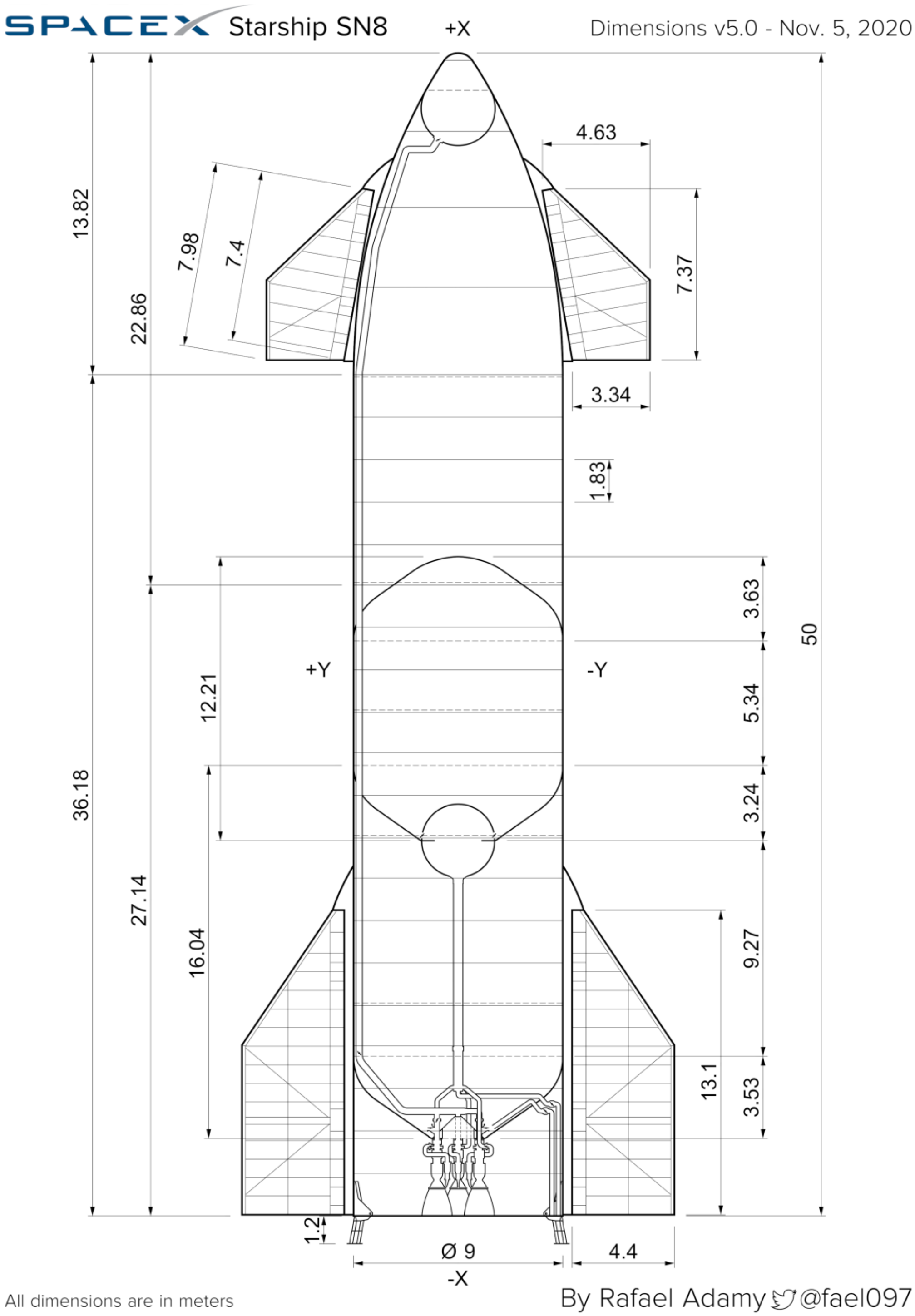

Figure 15. Unofficial dimensioned schematic of Starship SN 8. Courtesy of Rafael Adamy [44]

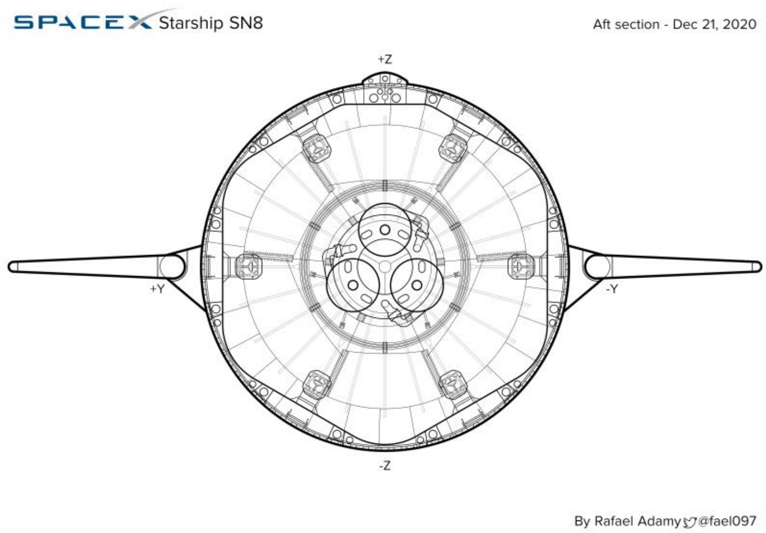

Figure 16. Unofficial aft schematic of Starship SN 8. Courtesy of Rafael Adamy [44]

Figure 17. Image from SpaceX Starship SN15 High-Altitude Flight Test webcast showing flap deployment neutral position of about 45-degrees. Courtesy of SpaceX [16]

V. References

[1] U.S. Standard Atmosphere, Washington, D.C.: U.S. Government Printing Office, 1976.

[2] D. R. Williams, "Earth Fact Sheet," NASA Goddard Space Flight Center, 25 November 2020. [Online]. Available: https://nssdc.gsfc.nasa.gov/planetary/factsheet/earthfact.html. [Accessed 12 March 2021].

[3] SpaceX, "Starship Update," YouTube, 28 September 2019. [Online]. Available: youtu.be/sOpMrVnjYeY. [Accessed 12 April 2020].

[4] SpaceX, "Falcon User's Guide," April 2020. [Online]. Available: https://www.spacex.com/media/falcon_users_guide_042020.pdf. [Accessed 29 February 2021].

[5] @elonmusk, "The outer ring of booster engines (20 of 28) have no TVC actuators. Steering control comes from center 8 TVC & differential throttle of outer engines. By any measure, the high thrust variant of Raptor will probably have the high T/W of any engine ever.," Twitter, Inc., 1 September 2020. [Online]. Available: https://twitter.com/elonmusk/status/1300698052026683394. [Accessed 21 March 2021].

[6] SpaceX, "SpaceX - Starship," 2021. [Online]. Available: https://www.spacex.com/vehicles/starship/. [Accessed 2 May 2021].

[7] SpaceX, "Starship Users Guide," March 2020. [Online]. Available: https://www.spacex.com/sites/spacex/files/starship_users_guide_v1.pdf. [Accessed 8 April 2021].

[8] SpaceX, "First Private Passenger on Lunar Starship Mission," YouTube, 17 September 2018. [Online]. Available: youtu.be/zu7WJD8vpAQ. [Accessed 9 April 2020].

[9] NASA, "Space Shuttle Flights by Orbiter," NASA, 26 October 2012. [Online]. Available: https://www.nasa.gov/mission_pages/shuttle/launch/orbiter_flights.html. [Accessed 27 April 2021].

[10] R. W. Humble, G. N. Henry and W. J. Larson, Space Propulsion Analysis and Design, New York: McGraw-Hill, 1995, pp. 688, 690.

[11] Z. Rosenberg, "SpaceX aims big with massive new rocket," Flight Global, 15 October 2012. [Online]. Available: https://www.flightglobal.com/space/spacex-aims-big-with-massive-new-rocket/107434.article. [Accessed 24 April 2021].

[12] SpaceX, "Making Humans a Multiplanetary Species," YouTube, 27 September 2016. [Online]. Available: youtu.be/H7Uyfqi_TE8. [Accessed 10 April 2020].

[13] HVM, "Topic: Raptor Engine graphics/schematics and tips/tricks to make them," NASASpaceFlight.com, 5 February 2019. [Online]. Available: https://forum.nasaspaceflight.com/index.php?topic=47383.0. [Accessed 18 March 2021].

[14] J. O'Callaghan, "The wild physics of Elon Musk's methane-guzzling super-rocket," Wired, 31 July 2019. [Online]. Available: https://www.wired.co.uk/article/spacex-raptor-engine-starship. [Accessed 1 May 2021].

[15] T. Burghardt, "Starhopper successfully conducts debut Boca Chica Hop," NASASpaceFlight.com, 25 July 2019. [Online]. Available: https://www.nasaspaceflight.com/2019/07/spacex-resume-starhopper-tests/. [Accessed 19 April 2021].

[16] SpaceX, "Starship | SN15 | High-Altitude Flight Test," YouTube, 5 May 2021. [Online]. Available: https://youtu.be/z9eoubnO-pE. [Accessed 5 May 2021].

[17] NASA Kennedy Space Center, "Final Environmental Assessment for the SpaceX Starship and Super Heavy Launch Vehicle at Kennedy Space Center (KSC)," NASA, Kennedy Space Center, 2019.

[18] @elonmusk, "@PaigeANjax -340 F in this case. Deep cryo increases density and amplifies rocket performance. First time anyone has gone this low for O2.," Twitter, Inc., 17 December 2015. [Online]. Available: https://twitter.com/elonmusk/status/1318727162439008259. [Accessed 12 April 2021].

[19] @elonmusk, "~2.8m," Twitter, Inc., 23 May 2019. [Online]. Available: https://twitter.com/elonmusk/status/1131671141624365056. [Accessed 16 April 2021].

[20] B. McBride and S. Gordon, "Computer program for calculation of complex chemical equilibrium compositions and applications," NASA Reference Publication 1311, 1996.

[21] @elonmusk, "Yeah. Merlin, using a single pintle injector with cold, liquid propellant, needs a longer chamber than Raptor, which uses a large number of coaxial injectors, despite Merlin being less than half thrust. Propellant enters Raptor chamber as hot gas that's a," Twitter, Inc., 15 Jun 2020. [Online]. Available: https://twitter.com/elonmusk/status/1272649257837715456. [Accessed 23 March 2021].

[22] M. Martinez-Sanchez, "Lecture 14: Non-Equilibrium Flows," Massachusetts Institute of Technology, Cambridge.

[23] @SpaceX, "RVac and sea level Raptor with a human for scale," Twitter, Inc., 24 September 2020. [Online]. Available: https://twitter.com/SpaceX/status/1309317920636301313. [Accessed 10 May 2021].

[24] @elonmusk, "Yeah. Outer engines with much larger nozzles are fixed to airframe, inner engines have high gimbal range ~15 degrees," Twitter, Inc., 23 May 2019. [Online]. Available: https://twitter.com/elonmusk/status/1131620991166078976. [Accessed 6 April 2021].

[25] @elonmusk, "No, as the header tanks are quite small relative to main (~30 tons vs 1200 tons). Only uses very tip of the nose.," Twitter, Inc., 18 April 2020. [Online]. Available: https://twitter.com/elonmusk/status/1251386583846023171. [Accessed 9 April 2021].

[26] K. Dresia, S. Jentzsch, G. Waxenegger-Wilfing, R. D. Santos Hahn, J. Deeken, M. Oschwald and F. Mota, "Multidisciplinary Design Optimization of Reusable Launch Vehicles for Different Propellants and Objectives," *Journal of Spacecraft and Rockets*, pp. 1-13, 2021.

[27] C. Bergin, "Starship SN11 prepares to fly as SpaceX pushes for Orbital flight this summer," NASASpaceFlight.com, 15 March 2021. [Online]. Available: https://www.nasaspaceflight.com/2021/03/starship-sn11-spacex-orbital-flight-summer/. [Accessed 19 March 2021].

[28] @elonmusk, "That's our goal," 16 March 2021. [Online]. Available: https://twitter.com/elonmusk/status/1371995012825763842. [Accessed 3 April 2021].

[29] SpaceX, "CRS-18 Mission," YouTube, 25 July 2019. [Online]. Available: https://youtu.be/SlgrxVuP5jk. [Accessed 13 March 2021].

[30] SpacX, "Starship Orbital - First Flight FCC Exhibit," Federal Communications Commission, 13 May 2021. [Online]. Available: https://apps.fcc.gov/els/GetAtt.html?id=273481. [Accessed 13 May 2021].

[31] "Boca Chica Beach, TX," Lat-Long.com, [Online]. Available: https://www.lat-long.com/Latitude-Longitude-1330976-Texas-Boca_Chica_Beach.html. [Accessed 2 May 2021].

[32] Inter-agency Space Debris Coordination Committee, "IADC Space Debris Mitigation Guidelines IADC-02-01," United Nations Office for Outer Space Affairs, Vienna, 2007.

[33] Union of Concerned Scientitsts, "UCS Satellite Database," Cambridge, 2021.

[34] SpaceX, "Astronomy Discussion With National Academy of Sciences," SpaceX, 28 April 2020. [Online]. Available: https://www.spacex.com/updates/starlink-update-04-28-2020/index.html. [Accessed 28 April 2021].

[35] E. W. Lemmon, M. O. McLinden and D. G. Friend, "Thermophysical Properties of Fluid Systems," in *NIST Chemistry WebBook, NIST Standard Reference Database Number 69*, P. J. Linstrom and W. G. Mallard, Eds., Gaithersburg, MD: National Institute of Standards and Technology.

[36] MatWeb, "304 Stainless Steel," [Online]. Available: http://matweb.com/search/DataSheet.aspx?MatGUID=abc4415b0f8b490387e3c922237098da. [Accessed 30 April 2021].

[37] H. D. Curtis, Orbital Mechanics for Engineering Students, Oxford: Elsevier, 2014.

[38] L. E. George and L. D. Kos, "Interplanetary Mission Design Handbook: Earth-to-Mars Mission Opportunities and Mars-to-Earth Return Opportunities 2009-2024," NASA, Huntsville, 1998.

[39] @elonmusk, "Probably 5 or 6 with an optimized tanker, although filling up the ship in orbit isn't required for Mars, so 4 is possible," Twitter, Inc., 2 October 2020. [Online]. Available: https://twitter.com/elonmusk/status/1311907493182926849. [Accessed 24 April 2021].

[40] E. Musk, "Making Life Multi-Planetary," *New Space*, vol. 6, no. 1, pp. 2-11, March 2018.

[41] J. J. Sellers, Understanding Space: An Introduction to Astronautics, New York: McGraw Hill, 2004.

[42] D. Lineberry, "The 'Un-Flyable' Space Shuttle," NASA, 12 April 2011. [Online]. Available: https://www.nasa.gov/centers/langley/news/researchernews/unflyable.html. [Accessed 25 April 2021].

[43] @elonmusk, "There's no entry/braking burn & hypersonic angle of attack is ~70 degrees," Twitter, Inc., 9 November 2020. [Online]. Available: https://twitter.com/elonmusk/status/1325937554236043266. [Accessed 17 April 2021].

[44] R. Adamy, "SpaceX Starship SN8 Dimensions v5.0," 5 November 2020. [Online]. Available: https://forum.nasaspaceflight.com/index.php?topic=52396.msg2163890#msg2163890. [Accessed 13 March 2021].

[45] NASA History Division, Introduction to the Aerodynamics of Flight, Washington, D.C.: NASA Scientific and Technical Information Office, 1975.

[46] NASA Glenn Research Center, "Shape Effects on Drag," NASA, 13 May 2021. [Online]. Available: https://www.grc.nasa.gov/www/k-12/rocket/shaped.html. [Accessed 1 May 2021].

[47] @elonmusk, "Yes, but engines have a min throttle point where there is flameout risk, so landing on 3 engines means high thrust/weight (further away from hover point), which is also risky," Twitter, Inc., 4 February 2021. [Online]. Available: https://twitter.com/elonmusk/status/1357425717500407816. [Accessed 24 April 2021].

[48] @elonmusk, "Max demonstrated Raptor thrust is ~225 tons & min is ~90 tons, so they're actually quite similar. Both Merlin & Raptor could throttle way lower with added design complexity. Raptor preburner & Merlin gas generator flameout are what limit lower bound.," Twitter, Inc., 17 Aug 2020. [Online]. Available: https://twitter.com/elonmusk/status/1295553672454311941. [Accessed 28 April 2021].

[49] MATLAB, "MATLAB version 9.10.0.1602886 (R2021a)," The Mathworks, Inc., Natick, 2021.

[50] NASA, Wings in Orbit, Washington D.C.: U.S. Government Printing Office, 2010.

[51] @elonmusk, "Yeah. By default, engine with least lever arm would shut down if all 3 are good.," Twitter, Inc., 4 Februrary 2021. [Online]. Available: https://twitter.com/elonmusk/status/1357426784279609344. [Accessed 19 March 2021].

[52] E. Ralph, "SpaceX nails first Starship landing weeks after NASA Moon lander contract [updated]," Teslarati, 5 May 2021. [Online]. Available: https://www.teslarati.com/spacex-aces-first-starship-landing-sn15/. [Accessed 6 May 2021].